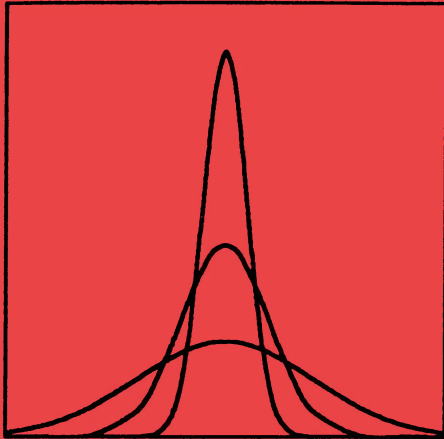


DEPARTMENT OF PHYSICS



95701680

PHYSICS

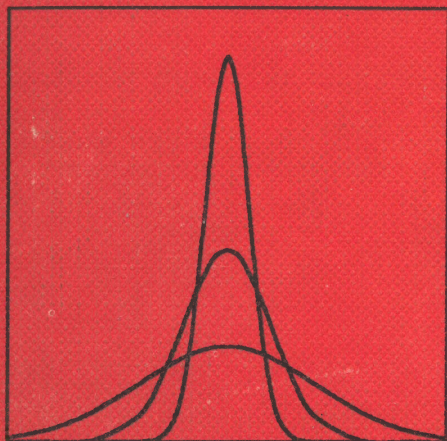
Only study guide for PHY103-8

(Practical work)



UNIVERSITY OF SOUTH AFRICA

DEPARTMENT OF PHYSICS



PHYSICS

Only study guide for PHY103-8

(Practical work)



UNIVERSITY OF SOUTH AFRICA

Laboratory number: 063

Exam Room: 5-178

Exam 10:00 - 13:00

criaba : p27
p28
p29

PHYSICS

Only study guide for PHY103-8

(Practical work)

Physics is phun!

Compilers

Dr I Basson

Prof E C Reynhardt

Revised by

Dr I Basson

period T of pendulum of length l :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Exp N^o:

3	4	5	6	8	9	1	2	

8/15 8/16 8/17 8/18 8/21 8/22 8/23 8/24

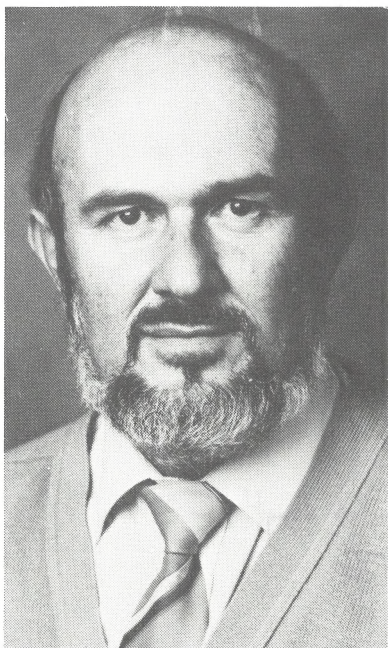
063

UNIVERSITY OF SOUTH AFRICA
PRETORIA

© 1992 University of South Africa

All rights reserved

Printed and published by the
University of South Africa
Muckleneuk, Pretoria



Prof E C Reynhardt



Dr I Basson

Contents

*Be able to draw circuits
Draw + interpret graphs
equations may be asked to be derived*

Preface	1
Introductory topics	5
1. Measurements and errors	7
2. Interpretation of results	40
3. Graphs	65
4. Planning an experiment and writing a report	93
5. Laboratory apparatus	105
Experiments in Mechanics	127
6. Linear air track (exp. 1)	129
7. Uniform circular motion (exp. 2)	154
8. Simple harmonic motion (exp. 3)	170
Experiments in Electromagnetism	185
9. Electrical measurements (exp. 4)	187
10. Charged particle in a uniform magnetic field (exp. 5)	204
11. Magnetic force on a current-carrying conductor (exp. 6)	219

*} very important
for exam*

Experiments in Optics and Modern Physics	231
~~12.~~ Experiments in geometrical optics (exp. 7)	233
13. Experiments in physical optics (exp. 8)	255
14. Photoelectric effect (exp. 9)	275
 Appendices	 287
A. Exponents and logarithms	289
B. The Greek alphabet	291
C. SI-units	292
D. Physical constants and conversion factors	293
E. Linear regression analysis	294
F. Demonstration experiments	295
G. Laboratory course	299

Preface

"This is indeed a mystery", remarked Watson, "What do you imagine that it means?"

"I have no data yet. It is a capital mistake to theorise before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts".

The Memoirs of Sherlock Holmes

by A Conan Doyle

This module deals with certain practical aspects of the 1st level Physics modules. The main activity of the module is therefore the compulsory laboratory course which will be presented at Unisa in Pretoria during August. However, this does not imply that you will only pay attention to this module during the laboratory course. You are expected to submit assignments based upon the contents of the Study Guide before the laboratory course starts. Admission to the laboratory course will be based on your achievements in these assignments.

Detailed information regarding the assignments is given in Tutorial Letter 101.

Information about the activities during the laboratory course, rules which must be adhered to and procedures to be followed, are listed in Appendix G in this Study Guide. You are advised to study it attentively before you attend the laboratory course.

The objectives of this Study Guide are as follows:

1. *To provide the required background knowledge* for the collection, interpretation and documentation of experimental data. In the section Introductory Topics aspects such as the handling of experimental errors, the drawing of graphs, laboratory apparatus and the writing of a scientific report are discussed.
2. *To discuss the experiments* that you will execute. The underlying theory of each experiment is discussed briefly, the apparatus is introduced and guidelines for the execution of the experiment are given.
3. *To test and improve your understanding of the relevant theory.* Hopefully the problems appearing at the end of each chapter will play an important role in this respect. You are advised to solve as many of these problems as possible. The following quotation by the German mathematician David Hilbert (1862–1943) should encourage you: "It is by the solution of problems that the strength of the investigator is hardened; he finds new methods and new outlooks and gains a wider and freer horizon".
4. *To provide important additional information.* Important physical constants (Appendix D), SI-units (Appendix C) and a discussion of logarithms (Appendix A) are examples. A number of demonstration experiments which are situated in the corridor of the Physics Department, are discussed in Appendix F.

If you come across a concept or theory in this study guide, which is unfamiliar to you or which you don't understand, please *LOOK IT UP!* Do so in the textbook of your other Physics modules and remember the library !

If you complete the assignments satisfactorily and gain admission to the laboratory course you should be well-equipped to complete the laboratory course successfully and to benefit optimally from it.

The objectives of the laboratory course are the following:

1. To *demonstrate theoretical ideas* in Physics. Seeing something demonstrated in practice is often a great help in understanding it. Sir Isaac Newton said : "In learning the sciences, examples are of more use than precepts".
2. To *provide familiarity with apparatus*. During this course you will handle a number of simple instruments and the experience you gain in the process should prove useful. However, the range of instruments you could conceivably work with is enormous. The main aim is therefore to train you to use instruments in general. Because the microcomputer plays an ever increasing role not only in Physics, but in our daily lives as well, we would also like to afford students the opportunity of gaining practical experience in the use of a microcomputer. A number of microcomputers will be available for the treatment of experimental data. Details are given in Appendix E.
3. To provide a training in *how to do experiments*. In this respect it is important to keep the following in mind:
 - Plan an experiment.
 - Be aware of systematic errors in methods and instruments and try to eliminate such errors.
 - Analyse the results and draw conclusions correctly.
 - Determine the accuracy of the final result.
 - Record the measurements and calculations accurately, clearly and concisely.

4. To offer you the opportunity *to meet your lecturers*. Usually all lecturers are involved in the laboratory course and you will have ample opportunity to discuss problems, including those related to the theoretical modules, with them.
5. To avoid confusion about notation. Different textbooks use different notations. In this study guide we use the *decimal point* instead of the *comma*. The following notation will be followed and you are requested to use the same notation in your assignments :

x : quantity x or magnitude x

$\bar{x}$: average value of x

$|x|$: absolute value of x

$\vec{x}$: vector x

$\hat{x}$: unit vector x

We hope that you will find this module an enriching and enjoyable experience, that you will be a better observer, a more careful and precise thinker and a more deliberative problem solver when you have completed this module. You are invited to convey any comments or criticism concerning the presentation of this module to the lecturer in charge.

Success !

Introductory topics

Chapter 1

Measurements and Errors

When it is not in our power to determine what is true, we ought to follow what is most probable.

René Descartes (1596–1650)

1.1	Introduction	8
1.2	John versus Paul	8
1.3	The infinite experiment	13
1.3.1	Relative frequency distribution : normalization	13
1.3.2	Limiting frequency distribution	15
1.3.3	Histogram	17
1.3.4	Continuous distribution	20
1.4	Precision of the apparatus	23
1.4.1	Dispersion about the mean	23
1.4.2	Standard deviation	24
1.5	Normal distribution	28
1.6	Accuracy and precision	32
1.7	Kinds of errors	33
1.7.1	Systematic errors	33
1.7.2	Random errors	34
1.8	Questions	36

1.1 Introduction

Measurement is fundamental to the growth and application of science. But making a measurement is not enough. When we come to use the result of the measurement, we want to know if it is good enough for our purposes. A significant aspect of the work of an experimental scientist is concerned with identifying sources of error, reducing their effects and assessing the reliability of the final result. When we do an experiment, we must recognise that there are errors and we must know how to deal with them.

1.2 John versus Paul

Two students were asked to time the oscillations of the same pendulum. John said the period was 1.4 s, Paul said it was 1.53 s. Who was to be believed - John because he seemed a dependable sort of person, or Paul because he gave the result to two decimal places?

A golden rule in experimental physics is to make sure how much information really comes from the experimental results and how much from preconceptions about the experiment or the person carrying it out. What we have to decide here is not only the period of the pendulum, but also the reliability of the measurements made. A single numerical result cannot by itself give both pieces of information. At least two numbers are required if an experiment is to give a result and a measure of its reliability, although, as we shall see later, it is much more useful to include yet a third.

Back to John and Paul. To decide who is "correct", it would be more satisfactory to ask them to *repeat* their measurements. The first five results from John were 1.4, 1.7, 1.4, 1.6 and 1.7 s and those from Paul 1.53, 1.50, 1.51, 1.52 and 1.51. We can compare these results best by plotting them as *frequency distributions*, with the measurement value as abscissa (on x-axis) and the number of times each value is found as ordinate (on y-axis) (see figures 1.1 and 1.2). The frequency f of a value is therefore defined as the number of times the value occurs.

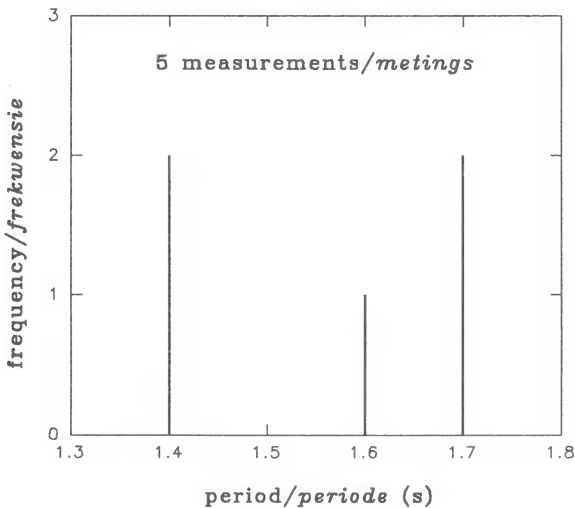


Figure 1.1 Frequency distribution of John's results after 5 measurements.

This gives a very different view of the matter. For one thing, both experimenters revised their original estimates. Both decided to give the *arithmetic average* or *mean* of his results. The arithmetic mean $\bar{x}$ of any set of values $x_1, x_2, x_3, \dots, x_n$ is defined as

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n} = \frac{\Sigma x}{n} \quad (1.1)$$

(Read $\sum_{i=1}^n x_i$ as "sigma x_i for i equals 1 to n "; sigma means the "sum of" and is abbreviated as Σx).

If the set of values occurs with frequencies $f_1, f_2, f_3, \dots, f_n$, the arithmetic mean is given by

$$\bar{x} = \frac{f_1 x_1 + f_2 x_2 + \dots + f_n x_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i} = \frac{\Sigma fx}{\Sigma f} \quad (1.2)$$

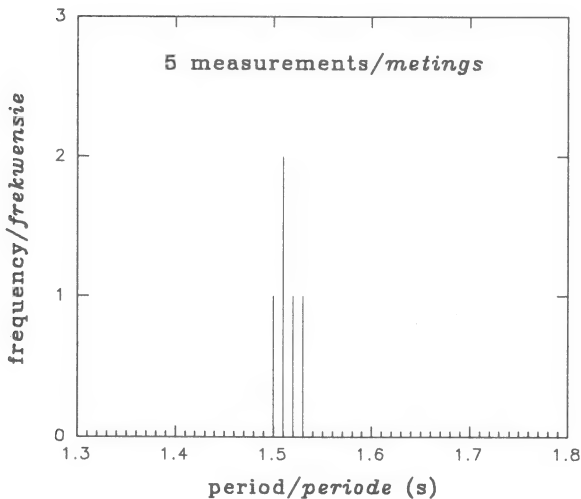


Figure 1.2 Frequency distribution of Paul's results after 5 measurements.

Example 1.1

The length (in mm) of a bar was measured five times and the following values obtained :

x (mm) : 52, 54, 51, 56, 53

From equation (1.1) the mean is

$$\begin{aligned}\bar{x} &= \frac{(52+54+51+56+53)}{5}\text{mm} \\ &= 53 \text{ mm.}\end{aligned}$$

When the bar was measured ten times, the following data were obtained :

$x(\text{mm})$:	51	52	53	54	56
f :	1	3	3	2	1

From equation (1.2) the mean is

$$\begin{aligned}\bar{x} &= \frac{(1 \times 51) + (3 \times 52) + (3 \times 53) + (2 \times 54) + (1 \times 56)}{1+3+3+2+1}\text{mm} \\ &= 53 \text{ mm.}\end{aligned}$$

John then said the period was $(1.4+1.7+1.4+1.6+1.7)/5 = 1.56$ s in stead of 1.4 s and Paul said it was $(1.53+1.50+1.51+1.52+1.51)/5 = 1.514$ s in stead of 1.53 s. Secondly, we have an objective way of comparing the two experiments. Paul's measurements are closely bunched together, 1.50 s to 1.53 s, while John's measurements spread from 1.4 s to 1.7 s. Paul's measurements look more accurate than John's, but we cannot be very precise about what we mean by "more accurate", since the two distributions are sufficiently different for Paul's value of 1.514 s to be preferred to John's 1.56 s.

However, John was not prepared to take this lying down. He put several hours' work in and came back with 500 measurements and a mean value of 1.5326 s. His frequency distribution is now as illustrated in figure 1.3.

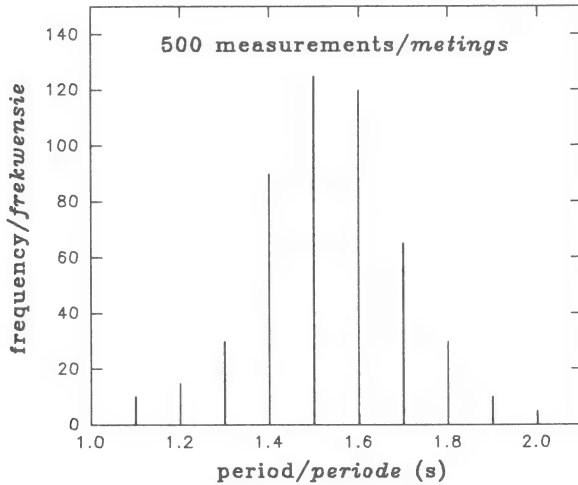


Figure 1.3 Frequency distribution of John's results after 500 measurements.

This is still a much broader one than Paul's and in this sense is inferior. But should not some credit be given to the fact that it represents 500 measurements instead of just 5? Whom should we believe at this stage - John or Paul? And what is the correct value for the period? In what follows we shall try to show how a systematic way of analysing experimental results will help us to answer these questions. We shall see that there is no "correct" answer to any problem of this type. However, there are usually good grounds on which we can agree about the "best" answer, and even if this agreement cannot be reached, a clear method of looking at the problem will at least enable us to see just what we are disagreeing about.

1.3 The infinite experiment

A question that arises from the previous section is how to express the importance of repeating measurements. We would like to find a *numerical* way of comparing John's 500 measurements with the 5 of Paul. We can follow an experiment through by plotting the frequency distribution as more and more measurements are made. Figures 1.1 and 1.3 show it at the 5- and 500-measurement stage of John's experiment. It is *changes* in this distribution that we must try to relate to the number of measurements made.

1.3.1 Relative frequency distribution : normalization.

One way in which the distribution changes is that the ordinates become larger as the number of measurements increases, but it is not obvious to compare the results of 5 measurements with those of 500 if we use the same scale. A better way of presenting the results is by means of *relative frequency distributions*, where the ordinates are not the actual or absolute number of times a measurement is recorded, but the *ratio of this number of the total number of measurements* made at that stage. The relative frequency r_i with which value x_i occurs, is therefore given by

$$r_i = \frac{f_i}{\sum_1^N f_i} \quad (1.3)$$

Figures 1.1 and 1.3 converted into relative frequency distribution form, would then be as shown in figures 1.4 and 1.5 (take notice of the ordinate values !).

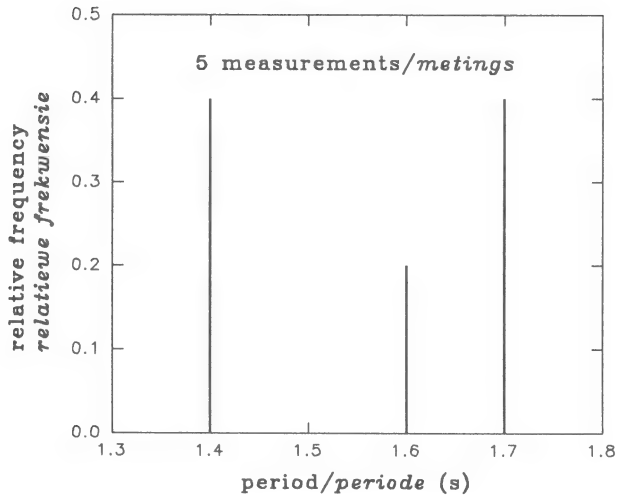


Figure 1.4 Relative frequency distribution of John's results after 5 measurements.

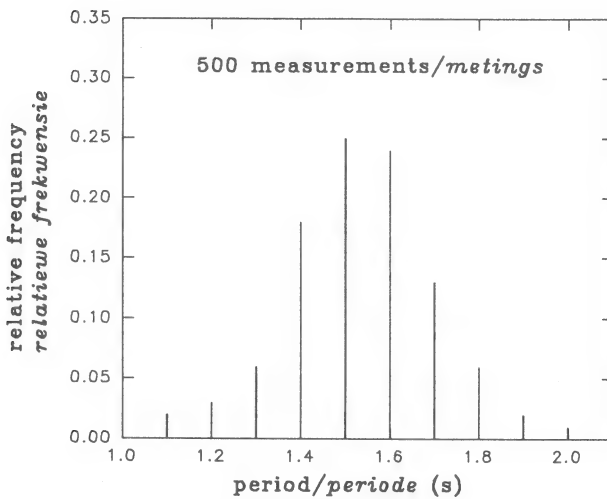


Figure 1.5 Relative frequency distribution of John's results after 500 measurements.

When presenting results as a relative frequency distribution, it is essential to record the total number of measurements made. Both the distribution and this number are required for the proper interpretation of an experiment.

An important feature of relative frequencies is that their sum R is equal to *one*

$$\begin{aligned} R &= \sum_{i=1}^n r_i \\ &= \sum_{i=1}^n \left[\frac{f_i}{\sum f_i} \right] \\ &= 1. \end{aligned} \tag{1.4}$$

A distribution with this property is said to be *normalized*, so that changing a frequency distribution from absolute to relative form, is equivalent to *normalizing* the distribution.

1.3.2 Limiting frequency distribution.

Figures 1.4 and 1.5 show that even when they are normalized, frequency distributions fluctuate as the total number of measurements increase. It is an experimental fact that for most experiments these fluctuations are quite large and erratic for small numbers of measurements. We find, for example, that the distribution for 5 measurements will usually look rather different from that for 10 measurements. However, as the numbers increase the fluctuations decrease. There are differences, but much less marked, between the distributions for 50 and 100 measurements, and when we come to compare 500 and 1000 measurements the difference between their normalized frequency distributions will be negligible.

We usually have no way of predicting exactly what the fluctuations will be, only that the distribution settles down to a more and more definite shape as the number of measurements increases. This we summarize by saying that there is a *limiting frequency distribution* for the *infinite experiment*. This limiting distribution is the basis of the whole of our subsequent discussion.

It is very important to realize that the existence of this limiting distribution is nearly always an assumption. We can never really test it satisfactorily. If an experimentalist keeps on and on making a measurement he may become tired and careless, or the apparatus will begin to wear out, both of which will tend to broaden the frequency distribution. Or it may be that the quantity that is being measured is itself varying with time - room temperature, for example - so that the frequency distribution will drift up or down as the measurements are repeated. The assumption implies that with sufficient ingenuity we could make the measurements as many times as we like *without any change* in the properties of the apparatus, the observer, or the quantity that is being observed - a matter which clearly cannot be put to the test.

So our whole analysis of experimental errors is built upon an assumption, and this may seem to be a very uncertain and unsatisfactory basis for an important part of science. However, we should not feel futile about this. All scientific theories start from some assumptions, although they are often not clearly stated or understood. There is an honourable and well-tested way of proceeding in these circumstances. We simply develop the theory to the extent that it makes predictions that can be tested against experimental results or other theories. If discrepancies appear we look back along our line of argument, examining each assumption as we

come to it, to see how necessary it was to enable us to develop the theory and to see what possible changes could remove the discrepancy.

It is safe to say that any experiment occurring in a school or undergraduate course, is extremely unlikely to cast doubt on our initial assumption of a limiting frequency distribution and that it should be possible to resolve the discrepancies that do occur, in a less deep and philosophical way than the revision of the basic assumption.

1.3.3 Histogram.

If x is a continuous variable, that is one which would have any value (perhaps within a restricted range), a histogram is useful in representing the values. To construct a histogram, the range of measured values is divided into a set of equal intervals and then the number of times a measurement occurs in each interval is counted. It is important that there should be no doubt as to where the boundaries of the intervals are and that there should not be overlapping or gaps between successive intervals. For example, if we decide on intervals with end points $x_0, x_1, x_2, \dots, x_n$ the intervals are

$$\begin{aligned} x_0 &\leq x \leq x_1 \\ x_1 &< x \leq x_2 \\ &: \\ &: \\ &: \\ x_{n-1} &< x \leq x_n \end{aligned}$$

(take notice off the " $\leq$ " and "<" signs !), with $f_1, f_2, \dots, f_n$ the frequencies and n the total number of measurements. The histogram is illustrated in figure 1.6.

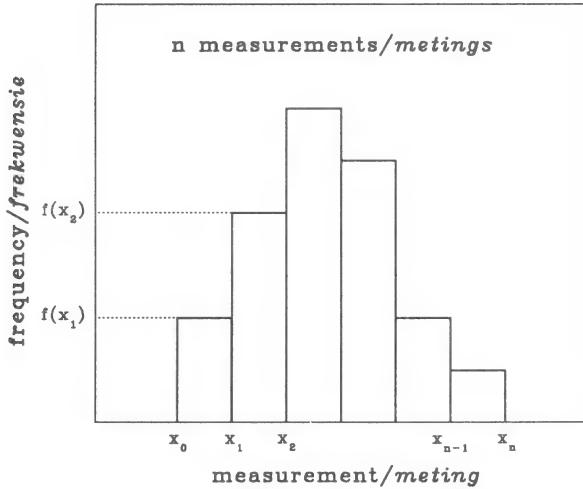


Figure 1.6 General representation of a histogram.

As explained previously, *relative* frequencies are more general and informative. Therefore they constitute a *normalized histogram* of the results, when these values are shown as rectangles based on each measurement interval. In this case the area under the histogram is also *one* (see equation 1.4).

Example 1.2

In table 1.1 an example of such a count is given after the length of a screw has been measured a number of times. Figure 1.7 shows the histogram obtained from the results.

Table 1.1

Intervals Length (mm)	Frequency f	Relative frequency r
9.9 – 10.1	1	0.03
10.1 – 10.3	3	0.09
10.3 – 10.5	7	0.21
10.5 – 10.7	9	0.27
10.7 – 10.9	6	0.18
10.9 – 11.1	5	0.15
11.1 – 11.3	2	0.06
	$\Sigma f = 33$	$R = \Sigma r = 1$

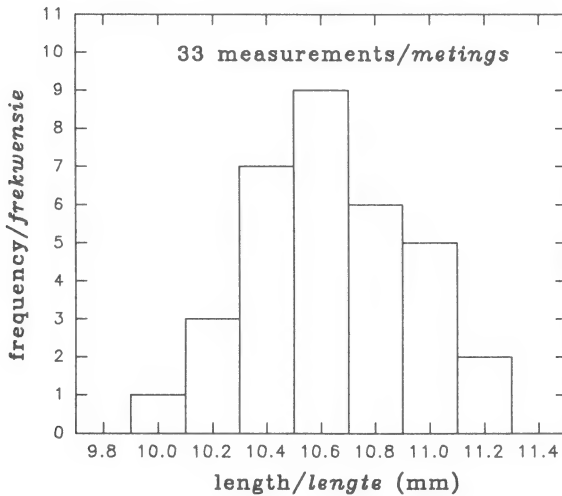


Figure 1.7 Histogram of the length of a screw measured 33 times.

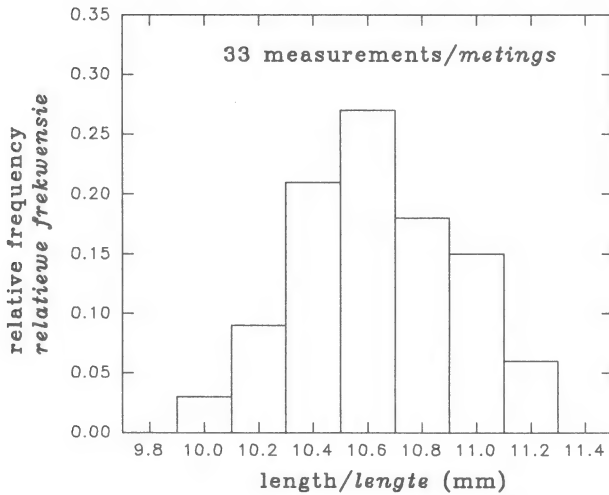


Figure 1.8 Normalized histogram of the length of a screw measured 33 times.

1.3.4 Continuous distribution

The cases of a *discrete frequency distribution* (as shown in figures 1.4 and 1.5) and a histogram (shown in figures 1.6 to 1.8), can be dealt with together in terms of a *continuous* distribution, if we imagine a *smooth curve* to be drawn through the outline of the diagrams (see figures 1.9 and 1.10).

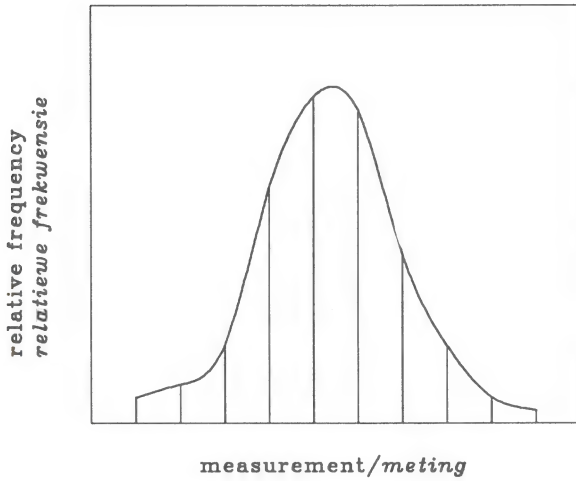


Figure 1.9 Continuous distribution associated with a discrete distribution.

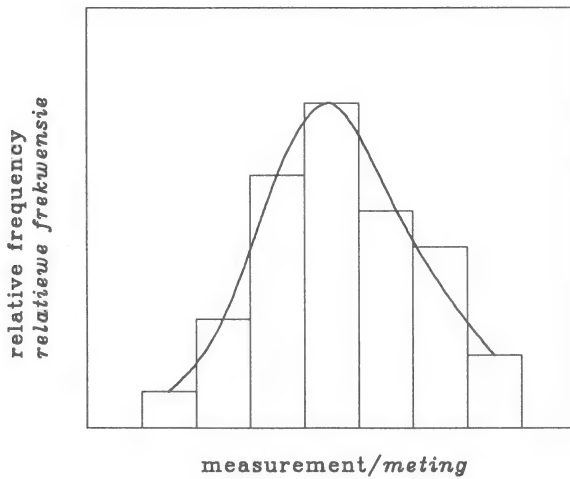


Figure 1.10 Continuous distribution associated with a histogram.

Given such a smooth curve, what is the value of the quantity we are trying to measure? This can be answered most easily when the limiting frequency distribution curve is *symmetrical* with a single peak as shown in figure 1.11.

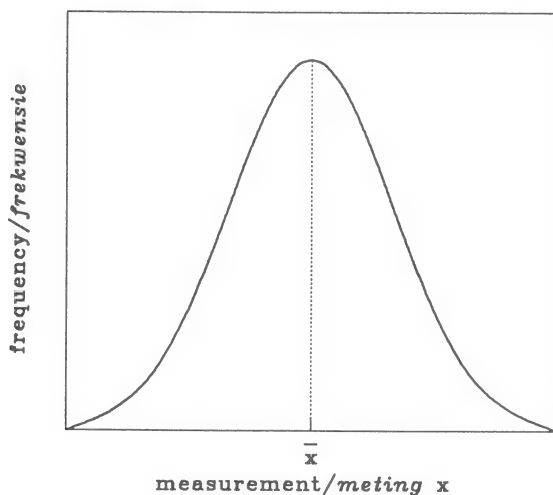


Figure 1.11 Symmetrical continuous frequency distribution.

The true value is then the value $\bar{x}$ at the central axis of symmetry of the curve. This value has unique properties. Among them are :

1. it is the *mean* of all the measurements
2. it is the *mode*, the value with the greatest frequency
3. it is the *median*, measurements below and above this value occur equally frequently, so that the ordinate through $\bar{x}$ divides the area under the curve in two equal parts.

In the case of a symmetrical distribution, the mean, mode and median all have the same value. If the curve is *not* symmetrical, the values are usually different, as illustrated in figure 1.12.

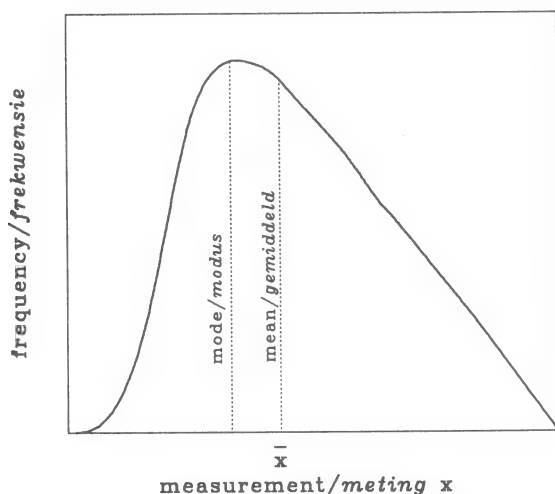


Figure 1.12 Asymmetrical continuous frequency distribution.

1.4 Precision of the apparatus

1.4.1 Dispersion about the mean

Often it is important to have a measure of the way in which the data are distributed or *dispersed* about the mean. In general, different sets of data have different means and different dispersions. The three numbers 24, 25 and 26 have a mean of 25 and so have the numbers 5, 25 and 45. The first group is within the limits indicated by 25 ± 1 while the second group is within the limits 25 ± 20 . The word "*range*" applied to a distribution of data means the difference between the

smallest and largest values. The range of the first group mentioned above is thus 2, while that of the second group is 40. It is a very simple measure of dispersion and has limitations. Another parameter often used as a measure of dispersion is the standard deviation.

1.4.2 Standard deviation

Suppose the same single number physical quantity is measured in two different experiments, involving different apparatus. If both limiting frequency distribution curves are single peaked and symmetrical, they should give the same value for the position of the peak. We should, therefore, expect something like figure 1.13. Note that the narrower curve (a) must have the taller peak since the areas under the two curves must be equal.

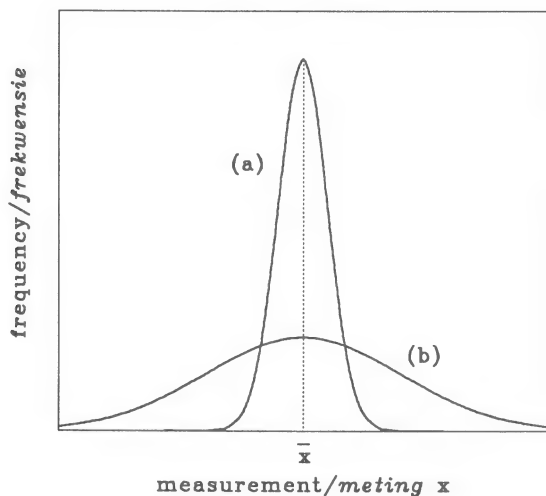


Figure 1.13 Limiting frequency distributions.

What, besides the common value $\bar{x}$, determines these curves? The role of the experimenter and the type of errors that can be made are discussed in §1.7. Since the only other factor determining them is the apparatus, the shape, as opposed to the position, must reflect in some way the properties of the apparatus. Of these its accuracy or precision is our concern, and although it is extremely difficult to define this in a comprehensive way, most of us would agree that in figure 1.13, (a) indicates a greater precision than (b). Comparing them, we see that a far greater proportion of the measurements obtained from experiment (a) (with the narrow distribution) will lie close to the true value, $\bar{x}$, than those from experiment (b) (with the broad distribution). This is roughly what we mean by saying that experiment (a) has a greater precision than experiment (b). The problem remains how to measure this precision in some simple, systematic, numerical way - that is, if possible, by another single number that determines the 'width' of the curve.

Any particular measurement x will differ from the true value $\bar{x}$ by an error, or deviation from the true value,

$$\delta = x - \bar{x} . \quad (1.5)$$

Large errors clearly correspond to broadly peaked curves, so some measure of the average size of the errors will give us the sort of number we are looking for.

There are many ways of obtaining a measure of the *average magnitude* of the errors. One of the simplest is the *standard deviation* and is defined as the *root-mean-square* deviation from the mean. Suppose $x_1, x_2, \dots, x_n$ is a set of values of parameter x . Their deviations from the mean $\bar{x}$ are given by equation (1.5) :

$$\delta_1 = x_1 - \bar{x}$$

$$\delta_2 = x_2 - \bar{x}$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$\delta_n = x_n - \bar{x}.$$

Thus, the standard deviation σ is

$$\begin{aligned}\sigma &= \left[\frac{\delta_1^2 + \delta_2^2 + \dots + \delta_n^2}{n} \right]^{\frac{1}{2}} \\ &= \left[\frac{\sum_{i=1}^n \delta_i^2}{n} \right]^{\frac{1}{2}}.\end{aligned}\quad (1.6)$$

If the values $x_1, x_2, \dots, x_n$ have frequencies $f_1, f_2, \dots, f_n$ then

$$\begin{aligned}\sigma &= \left[\frac{f_1 \delta_1^2 + f_2 \delta_2^2 + \dots + f_n \delta_n^2}{f_1 + f_2 + \dots + f_n} \right]^{\frac{1}{2}} \\ &= \left[\frac{\sum_{i=1}^n f_i \delta_i^2}{\sum_{i=1}^n f_i} \right]^{\frac{1}{2}} \\ &= \left[\frac{\sum_{i=1}^n f_i \delta_i^2}{N} \right]^{\frac{1}{2}}\end{aligned}\quad (1.7)$$

where

$$N = \sum_{i=1}^n f_i = \Sigma f. \quad (1.8)$$

From this it should be clear that the narrower the curve, the smaller σ is and the more precise the apparatus is. It must be emphasized that precision is too complex a concept to be completely described in this way, but for the purposes of this module, there is usually no better measure of precision of the apparatus, than the *standard deviation of the limiting frequency distribution*.

Example 1.3

The parameter x occurs with frequency f as shown below. Calculate the mean and the standard deviation.

$x:$	1	2	3	4	5	6	7	8	9	10
$f:$	3	4	10	18	25	50	20	7	2	1

The mean (from equation (1.2)):

$$\bar{x} = \frac{\sum_{i=1}^{10} f_i x_i}{\sum_{i=1}^{10} f_i}$$

$$= \frac{(3 \times 1) + (4 \times 2) + (10 \times 3) + (18 \times 4) + (25 \times 5) + (50 \times 6) + (20 \times 7) + (7 \times 8) + (9 \times 2) + (10 \times 1)}{3 + 4 + 10 + 18 + 25 + 50 + 20 + 7 + 2 + 1}$$

$$= \frac{762}{140}$$

= 5. (Note: read § 2.4 on significant figures to see why $\bar{x} = 5$ and not 5.44 !).

x	f	d	fd ²
1	3	-4	48
2	4	-3	36
3	10	-2	40
4	18	-1	18
5	25	0	0
6	50	1	50
7	20	2	80
8	7	3	63
9	2	4	32
10	1	5	25
Sum	140		392

The standard deviation (from equation (1.9)) is :

$$\begin{aligned}
 \sigma &= \left[\frac{\sum f d^2}{N} \right]^{\frac{1}{2}} \\
 &= \left[\frac{392}{140} \right]^{\frac{1}{2}} \\
 &= 2.
 \end{aligned}$$

1.5 Normal distribution

Having discussed the standard deviation, let's turn our attention once more to the *symmetrical distribution* (figure 1.11), which is also known as the *normal* or *Gaussian* distribution. (Other types of distributions are for example the binomial and Poisson distributions). The shape of the curve of a normal distribution is described by the equation

$$r = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\bar{x} - \bar{x})^2}{2\sigma^2}} \quad (1.9)$$

where r is the relative frequency, σ the standard deviation and $\bar{x}$ the mean. The shape of the curve is again illustrated in figure 1.14.

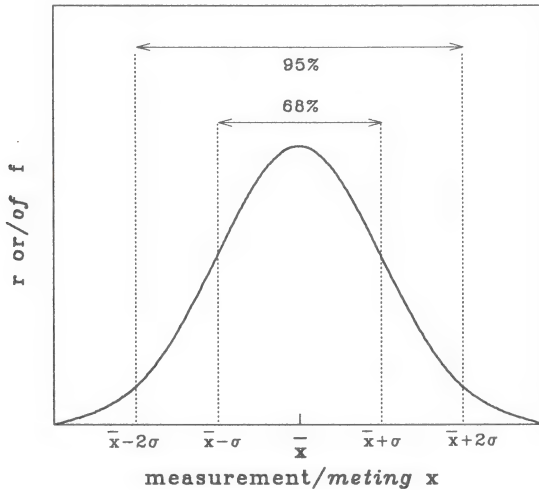


Figure 1.14 Normal distribution

The curve has the following features :

1. It is bell-shaped and symmetric about the mean $\bar{x}$.
2. For a large number of readings, 68% (or approximately $2/3$) of the readings will be within the range $\bar{x} \pm \sigma$ and 95% within $\bar{x} \pm 2\sigma$.
3. The constant $(1/\sigma\sqrt{2\pi})$ in equation (1.9) is chosen to ensure that the area under the curve is equal to 1.

The shape of the normal frequency curve for the same value of $\bar{x}$ and different values of σ , is shown in figure 1.15.

The normal frequency curve which fits any set of data best, can be found by determining the mean and standard deviation of the data (see next example).

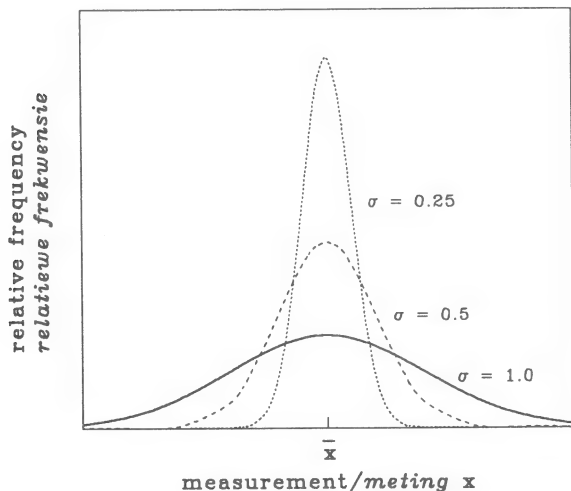


Figure 1.15 Shape of normal frequency curve for different values of σ .

Example 1.4

A student measures the width of a wooden beam at different positions along the length of the beam. The data tabulated are the number of times (f) each measurement of the width x is observed.

$x(\text{mm})$:	25	26	27	28	29	30	31	32	33	34
f :	1	5	17	49	85	52	25	11	4	1.

Find the equation of the normal frequency curve to fit the data.

x (mm)	f	fx (mm)	d (mm)	fd ² (mm ²)
25	1	25	-4	16
26	5	130	-3	45
27	17	459	-2	68
28	49	1372	-1	49
29	85	2465	0	0
30	52	1560	1	52
31	25	775	2	100
32	11	352	3	99
33	4	132	4	64
34	1	34	5	25
Sum	250	7304 mm		518 mm ²

The mean :

The standard deviation :

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$$\sigma = \left[\frac{\sum fd^2}{N} \right]^{\frac{1}{2}}$$

$$= \frac{7304 \text{ mm}}{250}$$

$$= \left[\frac{518 \text{ mm}^2}{250} \right]^{\frac{1}{2}}$$

$$= 29 \text{ mm.}$$

$$= 1.4 \text{ mm.}$$

Equation of the normal frequency curve :

$$r = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-\bar{x})^2}{2\sigma^2}\right]}$$

$$= \frac{1}{1.4 \sqrt{2\pi}} e^{-\left[\frac{(x-29)^2}{2(1.4)^2}\right]}$$

$$= 0.29 e^{-0.26(x-29)^2}$$

Figure 1.16 shows a plot of r as a function of x (dotted line) as well as the normalized histogram for the same data.

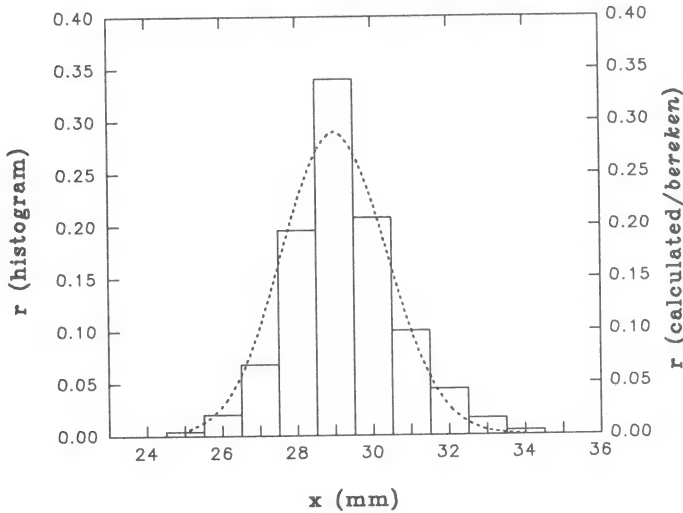


Figure 1.16 Normalized histogram and normal frequency curve for the data of example 1.4.

1.6 Accuracy and precision

Since it is impossible to measure the true value of a quantity, we must determine how much our measurement differs from the true value. By stating the *accuracy* of an observation we mean the closeness of agreement between the experimental value and the true value. Results of a measurement may also differ from the results of a replicate measurement made under apparently identical conditions. By *precision* we mean the

closeness of agreement between replicate experimental values. We thus could have a situation of a highly precise but inaccurate result. A *precise* measurement is one in which random errors are small; an *accurate* measurement is one in which both random and systematic errors are small (see § 1.7).

Accuracy - closeness to actual value : Small
Precision - Smaller random errors

1.7 Kinds of errors

1.7.1 Systematic errors

These are errors due to definite discoverable phenomena and can be controlled. The *accuracy* of a measurement can be increased by reducing the magnitude of these errors or by eliminating as many as possible.

There are various kinds of systematic errors depending on the type of apparatus or measurement. It is our task to recognise these errors and to try to eliminate them or to reduce their magnitudes. Repeating the measurement will not effect these errors since they are usually in "one direction" only.

Examples of systematic errors are the following:

1. Mistakes

Mistakes that occur frequently are the following : Following the wrong experimental procedure; misreading of a scale; incorrect recording of results; wrong calculation, etc. A mistake is a serious kind of error and should be avoided by using the necessary care and patience.

2. Personal errors

These errors are due to personal imperfections of the observer. His reactions on a signal may be too slow or his ability to distinguish between colours may be bad, etc.

3. Instrumental errors

All instruments have only a limited accuracy. It is the responsibility of the observer to find the limits of accuracy of any instrument before he uses it. The zero reading of an instrument may not be correct or screw instruments may have backlash. Most instrumental errors can be decreased by proper use.

4. Inherent errors

Examples of these errors are the following : the expansion of a measuring instrument with temperature; friction in mechanical experiments; change in room temperature during measurement; heat losses in calorimetric experiments; a variation in the emf of a battery; current losses in voltmeter, etc. You should attempt to recognise these errors and try to eliminate them by improving the experimental procedure.

1.7.2 Random errors

Random errors are caused by intrinsic and unpredictable fluctuations in the apparatus. After a long series of measurements these errors average to zero and consequently we can increase the *precision* of the measurement by increasing the number of measurements. The difference between systematic and random errors is illustrated in figure 1.17.

In the previous part of this chapter some elementary statistical methods of evaluating random errors were introduced. Note that the statistics deal only with *random errors*. Systematic errors cannot be evaluated by any statistical method.

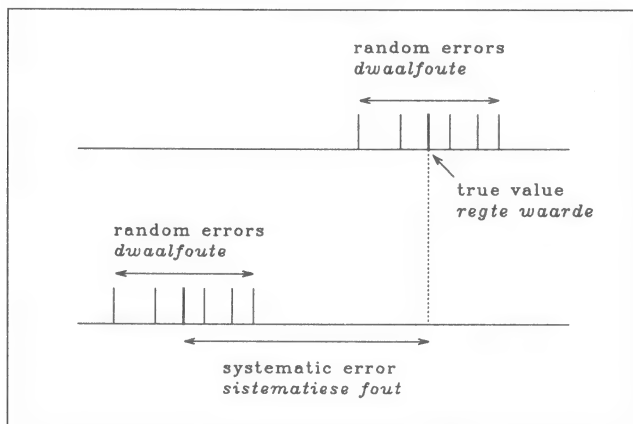


Figure 1.17 Illustration of the difference between systematic and random errors.

Example 1.5

Suppose that we know that a stone weighs 1.6921 newton. When the stone is weighed on a balance which has not been zeroed, the following readings are obtained: 1.6734, 1.6737, 1.6735, 1.6738 newton. These readings are very *precise*; the random error is only 0.0002 newton.

At the same time the readings are very inaccurate; the systematic error is 0.0185 newton. Obviously this large error cannot be eliminated by taking more readings. The scale should be zeroed to eliminate the systematic error.

1.8 Questions

- 1.1 The diameter of a wire is measured repeatedly at different places along its length. The measurements in millimeters are:

1.26, 1.26, 1.29, 1.31, 1.28, 1.27, 1.26, 1.25, 1.28, 1.32,
1.21, 1.27, 1.22, 1.29, 1.28.

Calculate the standard deviation of these readings and draw the frequency distribution.

- 1.2 Draw a

- histogram and
- relative frequency distribution for the data listed in Question 1.1.

- 1.3 The following measurements are obtained by measuring the time in seconds for one oscillation of a simple pendulum:

1.581 1.575 1.541 1.523 1.554 1.544 1.534 1.578 1.595
1.551 1.500 1.569 1.546 1.539 1.579 1.512 1.538 1.556
1.566 1.548 1.529 1.518 1.586 1.511 1.536 1.553 1.563
1.531 1.511 1.589 1.521 1.555 1.568 1.559 1.525 1.543
1.561 1.565 1.542 1.533 1.549 1.552.

61.09

- Draw a histogram of this set of measurements.

- Calculate the mean for the data. 1.5

- If the length of the pendulum is 60.0 ± 0.1 cm, what is the calculated value of the gravitational acceleration g ?

- 1.4 a) Find the mean of the lengths tabulated in Table 1.1.

- Draw the relative frequency distribution.

- Find the equation of the normal frequency curve and draw it on the same figure.

1.5 The variable x occurs with frequency f as shown :

x :	10	20	30	40	50	60	70	80	90
f :	35	52	74	95	81	62	48	32	21

- Find the mean and draw the frequency distribution.
- Draw a smooth curve to fit the data as closely as possible. Show how this curve is obtained.

1.6 The height h (in mm) of a barometer as recorded on 100 occasions were as follows :

h :	740	741	742	743	744	745	746	747	748	749	750
f :	1	5	9	23	20	17	12	6	4	2	1

- Find the mean, and
- draw the relative frequency histogram representing the heights.

1.7 Form a frequency table and plot a frequency distribution of the following data :

86	144	81	72	97	110	87	106	118
92	103	132	108	107	80	91	84	98
120	70✓	110	96	82	99	110	117	83
85	92	83	86	125	87	104	57	46
92	86	92	110	110	66	148	80	
109	95	107	99	94	108	89	118	
83	102	70✓	88	163	89	131	98	
104	130	88	75	91	97	76	94	
151	89	108	97	87	107	117	100	
74	108	106	103	104	80	134	92	
115	91	123	97	84	106	96	65	
100	87	86	110	102	96	94	161	

- Find the mean value.
- Draw a smooth curve to fit the data as closely as possible. Show how this curve is obtained.

- 1.8 Tabulate the relative frequencies for the following data and represent them graphically.

x :	15	16	17	18	19	20	21	22	23	24
f :	1	4	7	12	13	9	6	4	3	1

- 1.9 Find the standard deviation of the following:

15, 21, 19, 20, 18, 17, 22, 23, 16, 25.

- 1.10 Find the standard deviation of the set of numbers in Question 1.9 if their frequencies are

1, 11, 12, 14, 9, 6, 9, 5, 3, 1.

- 1.11 Calculate the standard deviation σ of the distribution consisting of the integers 1 to 5 inclusive. Samples are taken from these integers, each sample consisting of three different integers (10 samples in all) and a second distribution is formed from the means of these samples. If s is the standard deviation of this second distribution, prove that $s/\sigma = 1/\sqrt{6}$.

- 1.12 The following values of a quantity x were obtained experimentally :

x :	18	19	20	21	22	23	24	25	26
f :	1	5	8	12	10	7	4	1	2

- Calculate the mean,
- the standard deviation and
- draw the histogram for this distribution.

- 1.13 The following table gives the estimates of 100 individuals of the interpolation of the readings of a scale instrument:

Reading :	6.2	6.3	6.4	6.5	6.6	6.7	6.8
No. of individuals :	4	11	18	24	21	14	8

- Draw the relative curve and calculate
- the mean and
- the standard deviation from the mean.

1.14 The diameters of the spores of lycopodium can be found by an interference method. The results of an experiment are given below where $k = 5880$ when the diameters are measured in cm.

$k \times$ diameter :	14	15	16	17	18	19	20	21	22	23
No. of spores :	1	1	8	24	48	58	35	16	8	1

- Find the mean diameter of the spores, $\frac{\sum fx}{\sum f} = 18.5$
- the standard deviation, $\sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = 2.5$
- represent the data by a histogram and
- draw the corresponding normal frequency curve.

Chapter 2

Interpretation of Results

The more things change, the more they remain the same.

Michel de Montaigne (1533–1592)

2.1	Best estimate of the true value	41
2.2	Best estimate of precision	41
2.3	Accuracy of the mean value : standard error	43
2.4	Significant figures	46
2.4.1	General	46
2.4.2	Rounding off	47
2.4.3	Multiplication and division	47
2.4.4	Addition and subtraction	48
2.4.5	Number of significant figures	49
2.5	Combinations of measurements and errors	50
2.5.1	Addition	50
2.5.2	Subtraction	50
2.5.3	Scale factor	51
2.5.4	Linear combination	51
2.5.5	Product	52
2.5.6	General product	52
2.5.7	Exponents	53
2.5.8	Natural logarithms	53
2.5.9	Logarithms	54
2.5.10	Summary	54
2.5.11	Examples	55
2.5.12	Composite examples	57
2.6	Discussion of John and Paul's experiments	60
2.7	Questions	61

2.1 Best estimate of the true value

When using the limiting frequency curve, we are considering the results of an experimental measurement repeated an infinite number of times. In fact, we can make only a finite number of independent measurements, say n , and can make only an *estimate* of the true value, not a *precise* determination. Remember John? His first five measurements had a mean value of 1.56 s, but this is not necessarily the limiting or true value.

However, at any stage the mean $\bar{x}$ is the number we expect to approach the true value as n increases. We define the *best* estimate of the mean of the infinitely repeated measurement as the mean of the *finite* number of measurements (given by equation 1.2).

2.2 Best estimate of precision

The definition of precision also rests upon the notion of the infinitely repeated measurement. Once more we have to decide on the *best* estimate of this limiting number, which can be calculated from a finite number of measurements.

At first sight we might think that from n measurements we should first calculate the mean $\bar{x}$, then the deviation δ of each measurement from this and finally the standard deviation σ , which approaches the true value as n

increases. There is however a flaw in this argument which, although *unimportant* for *large* values of n , is *significant* for *small* values.

If we put $n = 1$, then $\bar{x} = x_1$, $\delta_1 = 0$ and $\sigma = 0$. If we take only *one* measurement, it must be the best estimate of the true value, but as we have seen initially when we compared John and Paul's first measurements, one measurement is not adequate to decide on the precision. The *least* information that will yield an estimate of precision is *two* independent measurements. In general, if n measurements $x_1, x_2, \dots, x_n$ are available, there are n ways of choosing x_i from among them, which leaves $(n-1)$ measurements for choosing a second value x_j . To provide for this bias, we define the *adjusted* standard deviation s by

$$\begin{aligned} s^2 &= \frac{n}{n-1} \sigma^2 \\ &= \frac{n}{n-1} \left[\frac{\delta_1^2 + \delta_2^2 + \dots + \delta_n^2}{n} \right] \quad \dots \text{ from (1.6)} \\ &= \frac{\delta_1^2 + \delta_2^2 + \dots + \delta_n^2}{n-1} \end{aligned}$$

$$\therefore s = \left[\frac{\sum_{i=1}^n \delta_i^2}{n-1} \right]^{\frac{1}{2}} \quad \text{To use } \delta_i \text{ (2.1)}$$

(compare equations 1.6 and 2.1 !).

This quantity s is almost equal to σ for large values of n , but gives a more reasonable interpretation of an experiment involving very few measurements. We shall therefore adopt s as the *best estimate* of σ , the precision of the apparatus.

2.3 Accuracy of the mean value : standard error

We now come to a rather more involved problem. We assume that the mean $\bar{x}$, approaches the true values, say $\bar{x}'$, in the limit as n becomes very large. Can we go further and say *how good* an estimate $\bar{x}$ of $\bar{x}'$ is?

If the frequency distribution has a narrow symmetrical peak, and therefore a small σ , the measurements will be close together and their mean will be near the true value $\bar{x}'$. If the peak is broad, the measurements will be more widely scattered and $\bar{x}$ is less likely to be near $\bar{x}'$. On the other hand, since we believe that $\bar{x}$ approaches $\bar{x}'$ as n increases, this means that by making n large enough, we can obtain a value of $\bar{x}$ as near as $\bar{x}'$ as we like. So even when the apparatus has poor precision (large σ or s), we can achieve any desired degree of accuracy in the estimated value by making a sufficient number of measurements.

If enough measurements are made for $\bar{x}$ to be near $\bar{x}'$, we should expect $\bar{x}$ not to fluctuate very much as the group of measurements by which it is defined, is varied. These fluctuations would show up if we repeatedly made groups of n measurements, each yielding a value for the mean

$$\bar{x} = (x_1 + x_2 + \dots + x_n)/n.$$

Rewrite this as

$$\bar{x} = \frac{x_1}{n} + \frac{x_2}{n} + \dots + \frac{x_n}{n}.$$

We can think of $\bar{x}$ as a quantity which itself is the sum of the measurements $x_1/n, x_2/n, \dots x_n/n$. The standard deviation of $\bar{x}$ then is (see § 2.5)

$$\sigma(\bar{x}) = \left[\frac{\sigma^2(x_1)}{n^2} + \frac{\sigma^2(x_2)}{n^2} + \frac{\sigma^2(x_n)}{n^2} \right]^{\frac{1}{2}} . \quad (2.2)$$

But since $x_1, x_2, \dots x_n$ are all measurements of the same quantity

$$\sigma^2(x_1) = \sigma^2(x_2) = \dots = \sigma^2(x_n) = \sigma^2(x).$$

Substitution in equation (2.2) gives

$$\sigma(\bar{x}) = \frac{\sigma(x)}{\sqrt{n}} . \quad (2.3)$$

The best estimate of $\sigma(x)$ is

$$s = \left[\frac{\delta_1^2 + \delta_2^2 + \dots + \delta_n^2}{n-1} \right]^{\frac{1}{2}} ,$$

therefore the best estimate of $\sigma(\bar{x})$ is

$$\begin{aligned} \Delta &= \left[\frac{\delta_1^2 + \delta_2^2 + \dots + \delta_n^2}{n(n-1)} \right]^{\frac{1}{2}} \\ &= \frac{s(x)}{\sqrt{n}} . \end{aligned} \quad (2.4)$$

In the equations above, $\sigma(\bar{x})$ is the *standard error* and its best estimate Δ , is the *adjusted standard error*.

The results of an experiment may be summarized in the form

$$x = \bar{x} \pm \Delta . \quad (2.5)$$

Note the following:

1. Δ is *not* a third quantity independent of s and $\bar{x}$, although it depends in a more explicit way on the number of measurements.
2. Δ involves *both* the intrinsic accuracy of the apparatus (σ) and the number of measurements (n), such that the accuracy of the mean increases as the intrinsic accuracy increases *and* as the number of measurements increases.
3. The improvement in accuracy with number of measurements goes only as $n^{-\frac{1}{2}}$. If for example, 10 measurements of the mean of a time yielded $\Delta = 0.1$ s, to improve this tenfold with the same apparatus to $\Delta = 0.01$ s, would require 1000 measurements. The same result could be obtained still with 10 measurements, if the precision of the apparatus were improved to 1/10 of its initial value.

It is important to recognize the *difference* between the best estimate of *precision* s and the best estimate of *error* Δ . As the number of measurements n increases, s *does not decrease*, it simply approaches the quantity σ which defines the precision of the experiment. Δ , however, *does decrease* as n increases, since it estimates the mean deviation of $\bar{x}$ from the true value $\bar{x}'$.

2.4 Significant figures

2.4.1 General

The precision of an experimental result is implied by the way in which the result is written. To indicate the precision, a number is written with as many digits as are meaningful. The number of *significant* figures (or digits) in a result is defined as follows :

1. The leftmost nonzero digit is the most significant digit.
2. If there is no decimal point, the rightmost *nonzero* digit is the least significant digit.
3. If there is a decimal point, the rightmost digit is the least significant digit, even if it is 0.
4. All digits between the least and most significant digits are counted as significant digits.

For example, the following numbers *each* have *four* significant digits :
1234, 123400, 123.4, 1001, 101.0, 0.001010.

If there is no decimal point, there are ambiguities when the rightmost digit is 0. The number 1010 for example, is considered to have only three significant digits even though the last digit might be physically significant. To avoid this ambiguity, it is better to write numbers in scientific notation. Our example of 1010 would be written as 1.01×10^3 if *three* and 1.010×10^3 if *four* digits are significant. Similarly 0.001010 is written as 1.010×10^{-3} and has *four* significant digits.

2.4.2 Rounding off

When quoting the result of an experiment, the number of significant figures given should be determined from the precision of the experimental data. In most cases it is necessary to drop *insignificant* digits from a number, especially after calculations have been performed.

To round off a number, truncate the number to the desired number of significant digits and treat the excess digits as a decimal fraction. Then

1. If the fraction is *greater* than $\frac{1}{2}$, increment the least significant digit.
2. If the fraction is *less* than $\frac{1}{2}$, do not increment.
3. If the fraction *equals* $\frac{1}{2}$, increment the least significant digit only if it is *odd*.

In this manner, the value of the final result is always within half the least significant digit of the original number. The reason for rule 3 is that in many cases the fraction equals either 0 or $\frac{1}{2}$ and consistently incrementing the least significant digit for a fraction of $\frac{1}{2}$ would lead to a systematic error. For example, 1.235 and 1.245 both become 1.24 when rounded off to *three* significant figures, but 1.2451 becomes 1.25.

2.4.3 Multiplication and division

In all computations involving multiplication and division, the factor with the least number of significant digits, determines the number of significant figures in the answer. To illustrate this, in

$$x = \frac{(8.2239)(2.7)(98.35)\pi^2}{2764}$$

$$= 7.797899\dots$$

the first three factors in the numerator have five, two and four significant digits respectively, π ($= 3.1415926\dots$) is known to an arbitrarily great accuracy and the denominator is known to four significant digits. The answer should therefore be rounded off to *two* digits, that is $x = 7.8$.

However, in *intermediate* steps of calculations, it is good to retain one or two *extra* digits to avoid introducing additional errors.

2.4.4 Addition and subtraction

The significant digits procedure used in addition and subtraction differs from that of multiplication and division. It is illustrated by the sum

$$\begin{array}{r} 45.76 \\ + \quad 0.123 \\ \hline 45.883 \end{array}$$

The 6 in the first number is somewhat uncertain and the next place is completely unknown. Accordingly the 3 in the sum is meaningless and the answer rounded to 45.88. The answer contains as many places relative to the decimal point as the "least accurate number" in the sum. In this example the least accurate number is 45.76 although it has four significant digits; 0.123 has only three significant digits but is more accurate in the sense meant here. The same ideas apply to subtraction.

2.4.5 Number of significant figures

A question that often occurs is : how many significant figures should be recorded? Here are some guidelines :

1. The accuracy to which an observation (length, time, mass, etc.) on the apparatus can be made, should give an indication of the number of significant figures to be recorded. For example, if the length of a book is measured with a millimeter ruler, the measurement is *accurate* to the nearest millimeter but an estimate of 0.5 mm can be made if the length falls in between 2 scale divisions. The length is for example then recorded as 204.5 mm, indicating 4 significant digits. If the length is *exactly* on the scale division at 204 mm, the measurement should be recorded as 204.0 mm, which still indicates the accuracy with 4 significant digits (the zero after the decimal point is meaningful in this case!).
2. For a *large* number of measurements, the relative accuracy of Δ gives a quantitative basis for deciding on how many digits to retain. This accuracy is given by $\sim 1/\sqrt{n-2}$. For example, if 100 measurements give values

$$\bar{x} = 17.386, \quad s = 1.226, \quad \Delta = 0.1226,$$

the relative accuracy of Δ is around 10% ($1/\sqrt{100-2}$). The third and fourth digits of s and Δ are then of no *practical* importance and the result can be written as

$$\bar{x} = 17.39 \pm 0.13 .$$

3. In any experiment involving 10 or fewer measurements, there is no point at all in giving s to more than *two* and Δ to more than *one* significant digit.

2.5 Combinations of measurements and errors

So far only errors on a single quantity were considered. It is however often the aim of an experiment to evaluate a parameter that is a function of several measured quantities, each with its own error. The error on such a parameter is called a *standard* error. The following rules on combining errors are valid for σ , s and Δ , although we will discuss it only for Δ .

2.5.1 Addition ($z = x + y$)

Suppose x and y are measured quantities with errors Δx and Δy (read "delta" x and "delta" y), written $x \pm \Delta x$ and $y \pm \Delta y$. Evaluating a quantity z from the sum of x and y

$$z = x + y, \quad (2.6)$$

the error Δz in z will arise partly from Δx and partly from Δy . The errors are *not* simply added, since sometimes the error on x may work in the opposite direction to that on y . The square root of the sum of the squares is taken as the error:

$$\Delta z = [(\Delta x)^2 + (\Delta y)^2]^{\frac{1}{2}} \quad (2.7)$$

2.5.2 Subtraction ($z = x - y$)

If z is the difference between x and y

$$z = x - y, \quad (2.8)$$

the error Δz is *also* given by equation (2.7). The magnitudes of the errors are still the same and they have the same chance of increasing or decreasing z .

2.5.3 Scale factor ($z = ax$)

If x is multiplied by a constant a

$$z = ax , \quad (2.9)$$

the error is multiplied by the same constant

$$\Delta z = a \Delta x . \quad (2.10)$$

Note that the percentage error on z is the same as on x .

2.5.4 Linear combination ($z = \alpha + ax + by + \dots$)

In the case of a linear combination

$$z = \alpha + ax + by + \dots \quad (2.11)$$

the error is given by

$$\Delta z = [a^2(\Delta x)^2 + b^2(\Delta y)^2 + \dots]^{\frac{1}{2}} . \quad (2.12)$$

Any of the scale factors $a, b, \dots$ may be negative. The sign affects the mean, but in the expression for the precision of the overall experiment, the contribution from each of the measurements, adds a positive amount.

2.5.5 Product ($z = xy$ or $z = x/y$)

The product of a value with another or with the inverse of the other is given by

$$z = xy \quad (2.13)$$

or

$$z = \frac{x}{y} . \quad (2.14)$$

In both cases the fractional increase in either x or y produces the same fractional increase in z . The errors Δx and Δy may lead to either an increase or a decrease in z and may cancel, therefore Δz is given by

$$\frac{\Delta z}{z} = \left[\left[\frac{\Delta x}{x} \right]^2 + \left[\frac{\Delta y}{y} \right]^2 \right]^{\frac{1}{2}} . \quad (2.15)$$

In equation (2.15), x and y are *independent* quantities. For cases like x^2 , see the paragraph on exponents.

2.5.6 General product ($z = \alpha x^a y^b$)

Given a general product

$$z = \alpha x^a y^b \quad (2.16)$$

the quantities involving errors, all occur in fractional or relative form. Hence

$$\frac{\Delta z}{z} = \left[a^2 \left[\frac{\Delta x}{x} \right]^2 + b^2 \left[\frac{\Delta y}{y} \right]^2 + \dots \right]^{\frac{1}{2}} \quad (2.17)$$

2.5.7 Exponents ($z = x^n$)

When z is given by

$$z = x^n \quad (2.18)$$

then the error follows from

$$\frac{\Delta z}{z} = n \frac{\Delta x}{x} \quad (2.19)$$

2.5.8 Natural Logarithms (e^x and $\ln x$)

For

$$z = \ln x \quad (2.20)$$

the error is given by

$$\Delta z = \frac{\Delta x}{x} \quad (2.21)$$

and for

$$z = e^x \quad (2.22)$$

the error is

$$\frac{\Delta z}{z} = \Delta x . \quad (2.23)$$

2.5.9 Logarithms (10^x and $\log x$)

For

$$z = \log x, \quad \Delta z = \frac{\Delta x}{2.3 \ x} \quad (2.24)$$

and for

$$z = 10^x, \quad \frac{\Delta z}{z} = 2.3 \ \Delta x \quad (2.25)$$

The factor 2.3 stems from $\ln 10 = 2.3$.

2.5.10 Summary

Table 2.1 Summary of combination of errors

Relation between x , y and z	Relation between Δx , Δy and Δz
$\left. \begin{array}{l} z = x+y \\ z = x-y \end{array} \right\}$	$(\Delta z)^2 = (\Delta x)^2 + (\Delta y)^2$
$z = ax$	$\Delta z = a \ \Delta x$
$z = a+ax+by+\dots$	$(\Delta z)^2 = a^2(\Delta x)^2 + b^2(\Delta y)^2 + \dots$
$\left. \begin{array}{l} z = xy \\ z = x/y \end{array} \right\}$	$\left[\frac{\Delta z}{z} \right]^2 = \left[\frac{\Delta x}{x} \right]^2 + \left[\frac{\Delta y}{y} \right]^2$
$z = a \ x^a \ y^b \dots$	$\left[\frac{\Delta z}{z} \right]^2 = a^2 \left[\frac{\Delta x}{x} \right]^2 + b^2 \left[\frac{\Delta y}{y} \right]^2 + \dots$
$z = x^n$	$\frac{\Delta z}{z} = n \frac{\Delta x}{x}$
$z = \ln x$; $z = \log x$	$\Delta z = \frac{\Delta x}{x}$; $\Delta z = \frac{\Delta x}{2.3 \ x}$
$z = e^x$; $z = 10^x$	$\frac{\Delta z}{z} = \Delta x$; $\frac{\Delta z}{z} = 2.3 \ \Delta x$

2.5.11 Examples

Work through these examples carefully.

1. Given: $A = 100 \pm 3$; $B = 45 \pm 2$ then

$$\begin{aligned}
 Z &= A + B \quad \text{and} \quad \Delta Z = [(\Delta A)^2 + (\Delta B)^2]^{\frac{1}{2}} \\
 &= 100 + 45 \quad \quad \quad = [(3)^2 + (2)^2]^{\frac{1}{2}} \\
 &= 145 \quad \quad \quad = 3,6 \\
 &\quad \quad \quad \simeq 4
 \end{aligned}$$

Answer: $Z = 145 \pm 4$ (remember significant digits!)

$$\begin{aligned}
 Z &= A - B \quad \text{and} \quad \Delta Z = 4 \\
 &= 100 - 45 \\
 &= 55
 \end{aligned}$$

Answer: $Z = 55 \pm 4$.

2. Given: $A = 25 \pm 2$; $c = 4$ (constant) then

$$\begin{aligned}
 Z &= cA \quad \text{and} \quad \Delta Z = c\Delta A \\
 &= (4)(25) \quad \quad \quad = (4)(2) \\
 &= 100 \quad \quad \quad = 8.
 \end{aligned}$$

Answer: $Z = 100 \pm 8$.

3. Given: $A = 156 \pm 7$; $B = 4.2 \pm 0.3$ then

multiplying A and B:

$$Z = AB \quad \text{and} \quad \frac{\Delta Z}{Z} = \left[\left(\frac{\Delta A}{A} \right)^2 + \left(\frac{\Delta B}{B} \right)^2 \right]^{\frac{1}{2}}$$

$$= (156)(4.2)$$

$$= 655.2 \quad \therefore \quad \Delta Z = (655.2) \left[\left[\frac{7}{156} \right]^2 + \left[\frac{0.3}{4.2} \right]^2 \right]^{\frac{1}{2}}$$

$$\simeq 660$$

$$= 55.2$$

$$\simeq 60.$$

Answer: $Z = 660 \pm 60$.

In the product the number of significant digits is determined by B which has 2. In the error, both ΔA and ΔB have only 1 significant digit, therefore ΔZ is rounded off to 1 significant digit.

Dividing A by B:

$$Z = \frac{A}{B} \quad \text{and} \quad \frac{\Delta Z}{Z} = \left[\left[\frac{\Delta A}{A} \right]^2 + \left[\frac{\Delta B}{B} \right]^2 \right]^{\frac{1}{2}}$$

$$= \frac{156}{4.2} \quad \therefore \quad \Delta Z = (37.1) \left[\left[\frac{7}{156} \right]^2 + \left[\frac{0.3}{4.2} \right]^2 \right]^{\frac{1}{2}}$$

$$= 37.1$$

$$= 3.1$$

$$= 37.$$

$$\simeq 3.$$

Answer: $Z = 37 \pm 3$.

4. Given: $A = 25 \pm 2$; $n = 3$ then

$$Z = A^n \quad \text{and} \quad \frac{\Delta Z}{Z} = n \frac{\Delta A}{A}$$

$$= (25)^3 \quad \therefore \quad \Delta Z = (3) \left(\frac{2}{25} \right) (15625)$$

$$= 15625$$

$$= 3750$$

$$\simeq 16000$$

$$\simeq 4000$$

Answer: $Z = 16000 \pm 4000$

$$= (1.6 \pm 0.4) \times 10^4.$$

5. Given: $A = 25 \pm 2$ then

$$\begin{aligned}
 Z &= \ln A & \text{and} & & \Delta Z &= \frac{\Delta A}{A} \\
 &= \ln(25) & & & &= \frac{2}{25} \\
 &= 3.2 & & & &= 0.08 \\
 & & & & &\approx 0.1
 \end{aligned}$$

Answer: $Z = 3.2 \pm 0.1$

6. Given: $A = 0.468 \pm 0.012$ then

$$\begin{aligned}
 Z &= e^A & \text{and} & & \frac{\Delta Z}{Z} &= \Delta A \\
 &= e^{0.468} & & & \Delta Z &= (0.012)(1,5968) \\
 &= 1,5968 & & & &= 0.01916 \\
 &\approx 1.60 & & & &\approx 0.02.
 \end{aligned}$$

Answer: $Z = 1.60 \pm 0.02$.

2.5.12 Composite examples

In the following examples Z is a given function of the independently measured quantities A , B , C and D . Calculate the value of Z and its standard error ΔZ .

$$\begin{aligned}
 1. \quad Z &= \frac{A(C^2 + D^{\frac{1}{2}})}{B} & \text{with} & & A &= 0.100 \pm 0.003 \\
 & & & & B &= 2.00 \pm 0.05 \\
 & & & & C &= 50.0 \pm 0.5 \\
 & & & & D &= 100 \pm 8
 \end{aligned}$$

$$\begin{aligned}
 \text{Solution: } Z &= \frac{(0.100)[(50.0)^2 + (100)^{\frac{1}{2}}]}{2.00} \\
 &= 125.5 \\
 &= 126.
 \end{aligned}$$

To find the error, group the quantities in the equation:

$$\begin{aligned}
 Z &= GH \quad \dots \quad G = \frac{A}{B} ; H = C^2 + D^{\frac{1}{2}} \\
 &= G(K+L) \quad \dots \quad K = C^2 ; L = D^{\frac{1}{2}}
 \end{aligned}$$

and determine the error for each group:

$$\begin{aligned}
 \text{For K: } K &= C^2 \quad \text{and} \quad \frac{\Delta K}{K} = 2 \frac{\Delta C}{C} \\
 &= (50.0)^2 \quad \therefore \quad \Delta K = \frac{2(0.5)(2500)}{(50.0)} \\
 &= 2500 \quad \quad \quad = 50.
 \end{aligned}$$

$$\begin{aligned}
 \text{For L: } L &= D^{\frac{1}{2}} \quad \text{and} \quad \frac{\Delta L}{L} = \frac{1}{2} \frac{\Delta D}{D} \\
 &= (100)^{\frac{1}{2}} \quad \therefore \quad \Delta L = \frac{1}{2} \left(\frac{8}{100} \right) (10) \\
 &= 10. \quad \quad \quad = 0.4.
 \end{aligned}$$

$$\begin{aligned}
 \text{For H: } H &= K+L \quad \text{and} \quad \Delta H = [(\Delta K)^2 + (\Delta L)^2]^{\frac{1}{2}} \\
 &= 2500 + 10 \quad \quad \quad = [(50)^2 + (0.4)^2]^{\frac{1}{2}} \\
 &= 2510 \quad \quad \quad = 50.
 \end{aligned}$$

$$\begin{aligned}
 \text{For G: } G &= \frac{A}{B} \quad \text{and} \quad \frac{\Delta G}{G} = \left[\left(\frac{\Delta A}{A} \right)^2 + \left(\frac{\Delta B}{B} \right)^2 \right]^{\frac{1}{2}} \\
 &= \frac{0.100}{2.00} \quad \therefore \quad \Delta G = (0.05) \left[\left(\frac{0.003}{0.100} \right)^2 + \left(\frac{0.05}{2.00} \right)^2 \right]^{\frac{1}{2}} \\
 &= 0.05 \quad \quad \quad = 0.002.
 \end{aligned}$$

The final answer is $Z = 126 \pm 6$. Please note that *no* rounding off has been done in the intermediate calculations. Only the final value Z and the final error ΔZ are rounded off.

Final answer: $Z = 46.1 \pm 0.3$.

2.6 Discussion of John and Paul's experiments

We are now able to summarize the information from any experiment as *three* numbers: the mean $\bar{x}$, which is the best estimate of the quantity we are trying to measure; the adjusted standard deviation s , which is the best estimate of the precision of the apparatus; and the adjusted standard error of the mean Δ , which is the best estimate of the overall accuracy of the experiment.

If we calculate $\bar{x}$, s and Δ at each stage of John and Paul's experiments, the results are summarized in table 2.2 .

Table 2.2 Summary of John and Paul's experiments.

Experimenter	Number of measurements	$\bar{x}$ (s)	s (s)	Δ (s)
John	1	1.4	-	-
Paul	1	1.52	-	-
John	5	1.56	0.15	0.07
Paul	5	1.514	0.010	0.005
John	500	1.533	0.163	0.007

Table 2.2 enables us to answer some of the questions we asked at the beginning of this study guide. We can say :

1. After one measurement John and Paul gave two independent and different results. No information was available on the precision of their apparatus or the accuracy of these results, so no comparison was possible.

2. After five measurements each, it appeared that Paul's apparatus was some fifteen times more precise than John's (0.01 s compared with 0.15 s). Paul's estimate of the true value of the period and of its standard error were 1.514 s and 0.005 s, which we shall write as $(1.514 \pm 0.005 \text{ s})$, compared with John's $(1.56 \pm 0.07 \text{ s})$. So the accuracy of Paul's estimate is more than ten times better than John's, and clearly, if one has to be chosen, it would be Paul's.
3. After John had increased his number of measurements to 500 the poor precision of his apparatus was confirmed. In fact it looks a little worse at this stage (0.163 s compared with the earlier value of 0.15 s), still somewhat over fifteen times less precise than Paul's. However, his hard work has had an effect, for his value of $(1.533 \pm 0.007 \text{ s})$ for the true value bears a standard error 0.007 s which, although rather greater than Paul's 0.005 s, is certainly not so different as to justify our disregarding his estimate completely in favour of Paul's. We might well feel that since the results are of comparable accuracy, and within 0.02 s of each other, that the true value is somewhere between the two, perhaps a little nearer Paul's because his is still the more accurate estimate of the two.

2.7 Questions

- 2.1 A motorist drives at a constant speed so that the reading on the speedometer is 40 km h^{-1} . The speedometer is assumed to be accurate to $\pm 2 \text{ km h}^{-1}$. At the end of the day he would like to know how far he has travelled, but unfortunately he forgot to look at the distance indicator when he set out. He reckoned that he had been driving for 4 hours, give or take a quarter of an hour. Estimate how far he travelled and assign an error to your result.

2.2 The volume of a rectangular block is calculated from the following measurements of its dimensions: 10.00 ± 0.10 cm, 5.00 ± 0.06 cm and 4.00 ± 0.04 cm. What is the error in the value of the volume of the block assuming

- a) the errors are independent,
- b) the errors are correlated such that they all push the estimate in the same sense?

2.3 The area of a circle is given by πr^2 where r is the radius. This can be written as $\text{Area} = (\pi) \times r$. Can we regard this equation as being similar in form to that of equation (2.13) where $z = \text{area}$, $x = \pi$ and $y = r$ and hence use equation (2.15) instead of equation (2.19) for determining the error in the area?

2.4 Consider the function

$$z = 2x + 3y$$

where

$$x = 13.36 \pm 0.03 \quad \text{and} \quad y = 6.27 \pm 0.05.$$

Calculate the value of z and the standard error.

2.5 Consider the function

$$z = (x/y^3)^{\frac{1}{2}}$$

where

$$x = 13.36 \pm 0.03 \quad \text{and} \quad y = 6.27 \pm 0.05.$$

Calculate the value of z and the standard error.

- 2.6 Consider the volume V of a sphere as calculated from a direct measurement of its diameter D :

$$V = \pi D^3/6 .$$

Suppose the value of D is measured as 3.04 cm and V computed to be 14.71 cm^3 . It is later discovered that the value of D has a systematic error of + 0.01 cm. What is the error in V and the correct value of V ?

- 2.7 A rectangular brass bar of mass M has dimensions a , b , c . The moment of inertia I about the axis in the centre of the ab face and perpendicular to it is

$$I = \frac{M}{12} (a^2 + b^2).$$

The following measurements are made:

$$M = 135.0 \pm 0.1 \text{ g}$$

$$a = 80 \pm 1 \text{ mm}$$

$$b = 10 \pm 1 \text{ mm}$$

$$c = 20.00 \pm 0.01 \text{ mm}.$$

What is the standard error in

- a) the density ρ of the material and
- b) the moment of inertia?

- 2.8 Round off the following quantities to three significant figures and write them in scientific notation:

- a) 276532.0
- b) 0.3729
- c) 4.6667
- d) 33.333
- e) 24.5558×10^3
- f) 0.000034567

2.9 How many significant figures are there in the final result for each of the following expressions?

- a) $(3.2)(8.67)/(3.008)$
- b) $(0.0002)(45.6)$
- c) $(2.0 \times 10^5)(3.777 \times 10^{-4})$
- d) $17.2 + 2.35 + 4.3333$
- e) $88.45 + 9.24 - 6.05043$
- f) $186.45 - 186.12$

2.10 Evaluate the following expressions in accordance with the rules for significant figures:

- a) $(3.28 \times 10^5) + (4.25 \times 10^7)$
- b) $(3.7 \times 10^6) + (2.91 \times 10^7)$
- c) $(1.91 \times 10^{-3}) - (1.7 \times 10^{-5})$
- d) $(5.48 \times 10^{-2})(2.6 \times 10^{-3})$
- e) $(6.77 \times 10^4)/(8.2347 \times 10^6)$
- f) $(2.1 \times 10^{-3})^2/(4.56 \times 10^{-2})$

Chapter 3

Graphs

Everything has its beauty, but not everyone sees it.

Confusius

3.1	Introduction	66
3.2	Why graphs are drawn	66
3.3	Straight line graphs	68
3.3.1	The straight line through the origin ($y = mx$)	68
3.3.2	The straight line not through the origin ($y = mx + c$)	69
3.4	Rewriting equations in straight line form	72
3.4.1	Equation $y = kx^n$	72
3.4.2	Equation $y = ke^{-cx}$	75
3.5	The best straight line	76
3.5.1	Estimating the best straight line	76
3.5.2	Points-in-pairs	76
3.5.3	Linear regression	80
3.6	Hints on how to plot graphs	84
3.7	Questions	88

3.1 Introduction

It is quite common in Experimental Physics that one variable or parameter is measured as a function of another variable. As an example we consider an object falling from rest in the gravitational field of the earth. You know already that the relationship between the speed v and the time t during which the object has been falling, is given by

$$v = gt + u. \quad (3.1)$$

Both v and t are variables while g , the gravitational acceleration, is a constant for a specific spot on the earth's surface. Similarly the initial speed is also a constant. Experimentalists would be interested in investigating how v varies as t increases. Which method we would employ to make the measurements is not relevant at the moment, but after the results have been tabulated we would like to present them graphically. In the next paragraph an attempt will be made to explain why results should be presented graphically.

3.2 Why graphs are drawn

A graph is a simple and convenient way of presenting a set of experimental results with a view to investigating it objectively and to draw reliable conclusions. The next example should illustrate the validity of this statement.

The experiment described in § 3.1 yields the following results :

t (s):	1	2	3	4	5	6	7	8	9	10
$v(\text{ms}^{-1})$:	10	50	30	44	56	64	75	85	95	105

We can now represent each set of values, for example 1 s and 10 m s^{-1} , as a point on a graph of speed versus time (figure 3.1).

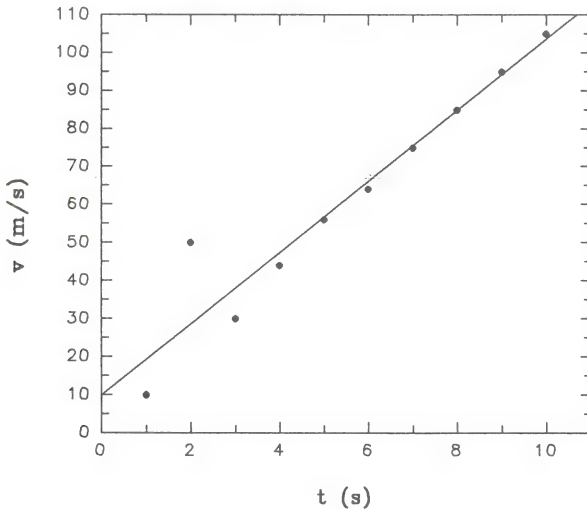


Figure 3.1 Graph of speed v as a function of time t .

This graph reveals the following facts about the data :

1. The speed is proportional to the time.
2. The data point corresponding to $t = 2 \text{ s}$ deviates considerably from the best straight line which has been drawn through the other data points. It should be considered a doubtful point. (When a point deviates a lot from the rest, this point can be ignored).
3. The accuracy of the measurements is lower if t is short.

4. If $t = 0$, $v \approx 6 \text{ m s}^{-1}$, i.e. the value of u , the initial speed.
5. The slope of the best straight line is approximately 10 m s^{-2} , i.e. the value of g at the place where the experiment has been executed.

Just by inspecting the data tabulated, these aspects cannot be observed. An additional advantage of a graph is that it makes it very easy for estimations to be made between the actual measured values. For example, although no measurements has been made at $t = 5.5 \text{ s}$, it follows from the graph that $v \approx 60 \text{ m s}^{-1}$. This process is called *interpolation*. *Extrapolation* enables us to estimate the value of a variable outside the range of the experiment. By extending the best straight line, it is predicted that the speed of the object after 12 s would be approximately 122 m s^{-1} . Extrapolation can yield misleading results and must be used with caution. Can you think of a good example in which extrapolation could yield misleading results?

3.3 Straight line graphs

Usually an attempt is made to draw a straight line graph because it is easy to draw and certain physical constants can be obtained directly from such a graph. Let us consider two mathematical equations which are used frequently to represent physical variables.

3.3.1 The straight line through the origin ($y = mx$)

If a variable y is directly proportional to another variable x , then

$$y = mx \quad (3.2)$$

where the constant of proportionality m is the slope of the graph when y and x are plotted as the variables on the ordinate (y axis) and the abscissa (x axis) respectively. The slope is given by

$$m = \frac{\text{increment in } y}{\text{corresponding increment in } x} \quad (3.3)$$

or

$$m = \frac{\Delta y (\text{measured in the units of the } y \text{ scale})}{\Delta x (\text{measured in the units of the } x \text{ scale})}.$$

Theoretically the line represented by equation (3.2) should pass through the origin since when $y = 0$, $x = 0$. This is shown by the line OA in figure 3.2. In practice an experimental line *may not pass through the origin due to a systematic error* in either x or y so that all the values of x or y are wrong by the same amount. This does not affect the value of m .

3.3.2 The straight line not through the origin ($y = mx + c$)

The line DB in figure 3.2 is also a straight line with slope m but it does not pass through the origin. This line is represented by

$$y = mx + c \quad (3.4)$$

where c is the distance from the origin to the point where the line or curve cuts an axis.

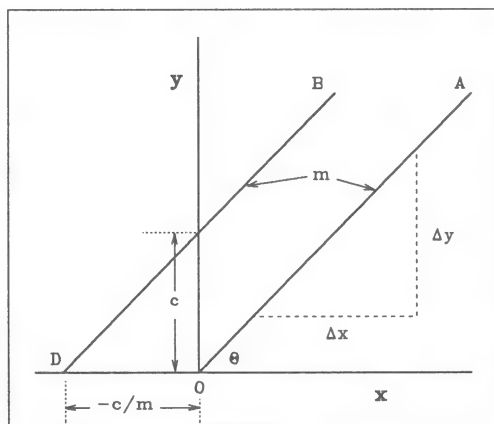


Figure 3.2 Straight line graph.

When DB intercepts the y axis, $x = 0$ and $y = 0 + c$. Hence c is the intercept on the y axis. When DB intercepts the x axis, $y = 0$ and $0 = mx + c$. Therefore $x = -c/m$ is the intercept on the x axis. From figure 3.2 it is clear that

$$m = \frac{-c}{\text{intercept on x axis}} \quad (3.5)$$

Example 3.1

The temperature dependence of a resistor is given by

$$R = R_0(1 + \alpha T). \quad (3.6)$$

A student wants to determine the constants R_0 and α and decides to measure the resistance R as a function of the temperature T . His results are listed below:

T (°C) :	31	41	51	61	71	81	91
R (Ω) :	5.35	5.55	5.71	5.90	6.07	6.28	6.44

How would you set about determining R_0 and α ? If you can't make any progress, you may look at the solution given below.

Equation (3.6) can be rewritten as

$$R = (R_0 \alpha)T + R_0. \quad (3.7)$$

You should realise immediately that it has the same form as equation (3.4) with

$$y = R_T, \quad m = R_0 \alpha \quad \text{and} \quad c = R_0.$$

If a graph (figure 3.3) of R versus T is drawn, a straight line graph should be obtained with slope equal to $R_0 \alpha$ and intercept along the R axis equal to R_0 .

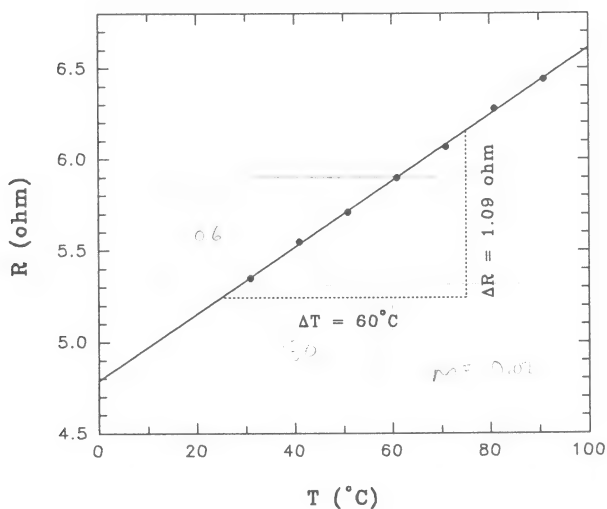


Figure 3.3 Graph of resistance as a function of temperature.

From the graph the slope is calculated as follows :

$$\begin{aligned} m &= \frac{\Delta R}{\Delta T} = \frac{1.09 \, \Omega}{60 \, ^\circ\text{C}} \\ &= 0.018 \, \Omega \cdot ^\circ\text{C}^{-1} \\ &= R_0 \alpha . \end{aligned}$$

From the intercept along the R axis it follows that

$$R_0 = 4.8 \, \Omega$$

and

$$\begin{aligned} \alpha &= \frac{m}{R_0} \\ \alpha &= \frac{0.018 \, \Omega \cdot ^\circ\text{C}^{-1}}{4.8 \, \Omega} \\ &= 3.8 \times 10^{-3} \cdot ^\circ\text{C}^{-1} . \end{aligned}$$

3.4 Rewriting equations in straight line form

Frequently it is necessary to rewrite non-linear equations in linear or straight line form.

3.4.1 Equation $Y = kX^n$

If the dependence of the variables measured in an experiment is given by

$$Y = kX^n \tag{3.8}$$

the equation can be written in the form

$$\begin{aligned}\log Y &= \log(X^n) + \log k \\ &= n \log X + \log k\end{aligned}\tag{3.9}$$

which is of the form

$$y = mx + c$$

with

$$y \equiv \log Y, \quad x \equiv \log X, \quad m \equiv n, \quad c \equiv \log k.$$

Plotting $\log Y$ as the ordinate and $\log X$ as the abscissa should yield a straight line graph with slope equal to n and intercept along the $\log Y$ axis equal to $\log k$.

(A discussion of logarithms appears in Appendix A).

Example 3.2

Situations do occur where the exact form of the dependence between variables is not known. In the case of a heated filament the relationship between the potential difference across the filament and the current is given by

$$V \propto I^n, \tag{3.10}$$

where n is the *power term*. The equation is now written as

$$V = DI^n, \tag{3.11}$$

where D is a constant. To obtain a straight line graph from a series of results for V and I , equation (3.11) has to be written in the form of equation (3.4). Take logs of both sides of the equation:

$$\begin{aligned}
 \log V &= \log(DI^n) \\
 &= \log D + \log I^n \\
 &= n \log I + \log D.
 \end{aligned}
 \tag{3.12}$$

If we compare equation (3.12) with (3.4) we see that

$$y \equiv \log V, \quad x \equiv \log I, \quad m \equiv n, \quad c \equiv \log D.$$

Plotting $\log V$ against $\log I$ yields a straight line graph shown in figure 3.4.

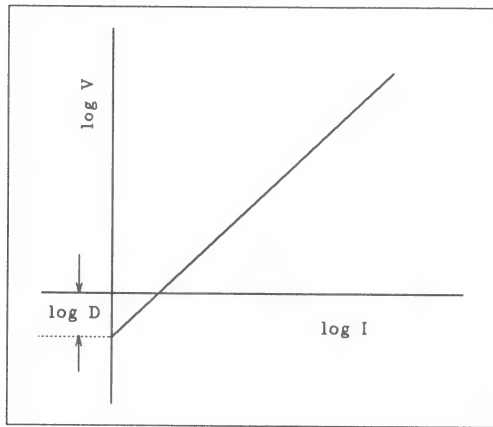


Figure 3.4: Graph of $\log V$ as a function of $\log I$.

Please note that the logarithm of a physical variable has no units. In figure 3.4 $\log V$ and $\log I$ do not have units although V and I have been measured in volt and ampere respectively.

3.4.2 Equation $Y = ke^{-CX}$

If the dependence of the variables measured in an experiment is given by

$$Y = ke^{-CX} \quad (3.13)$$

the equation can be rewritten as

$$\begin{aligned} \ln Y &= \ln k + \ln(e^{-CX}) \\ &= \ln k - CX \end{aligned} \quad (3.14)$$

which is of the form

$$y = mx + c$$

with

$$y \equiv \ln Y, \quad x \equiv X, \quad m \equiv -C, \quad c \equiv \ln k.$$

Plotting $\ln Y$ as the variable along the ordinate and X as the variable along the abscissa should yield a straight line graph with slope equal to $-C$ and intercept along the $\ln Y$ axis equal to $\ln k$.

(A discussion of the base of logarithms is given in Appendix A).

Example 3.3

Consider a radioactive decay process which is described by

$$N = N_0 e^{-\lambda t} \quad (3.15)$$

where N_0 is the number of atoms present at an arbitrary time ($t = 0$), N is the number left after the lapse of a further time

interval t and λ is the decay constant.

Taking logs to the base e , we obtain

$$\ln N = \ln N_0 - \lambda t. \quad (3.16)$$

Equation (3.16) has the same form as equation (3.4). If $\ln N$ is plotted as ordinate and t as abscissa the slope of the straight line graph is given by $-\lambda$. The intercept on the $\ln N$ axis is $\ln N_0$.

3.5 The best straight line

There are several methods that can be employed to determine the best straight line through experimental points on a graph.

3.5.1 Estimating the best straight line

In this case a transparent ruler is placed on the graph and shifted until it seems to represent the best straight line. It is a quick method but not very accurate. However, it is amazing how quickly practice leads to greater success.

3.5.2 Points-in-pairs

This method can be applied to a graph with *points equally spaced along the horizontal axis*. The method is illustrated in figure 3.5. The points are divided into two groups, one for low values and the other for high values of y . The points are then paired off, for example 1 and 7, 2 and 8, ... 6 and 12.

The difference between the values of y for each pair is then found:

$$d_1 = y_7 - y_1$$

$$d_2 = y_8 - y_2$$

$$\vdots$$

$$d_6 = y_{12} - y_6$$

The mean of these difference, $\bar{d}_y = (d_1 + d_2 + \dots + d_6)/6$, is calculated. If the equal distance between each of these pairs of points along the x axis is d_x , then the slope is given by

$$m = \bar{d}_y / d_x \quad (3.17)$$

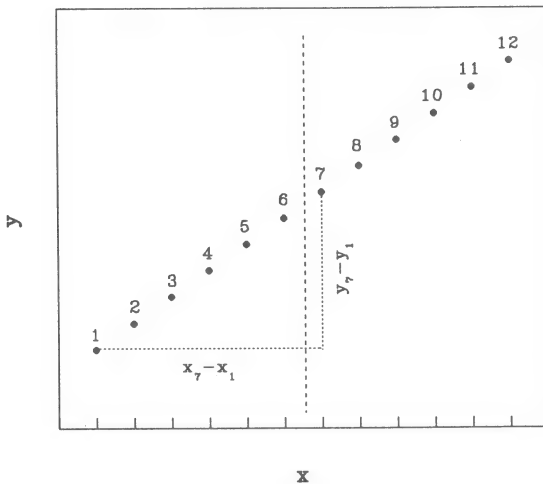


Figure 3.5 Illustration of the points-in-pairs method.

The mean values of x and y are then determined from:

$$\bar{x} = (x_1 + x_2 + \dots + x_n)/n \quad (3.18)$$

and

$$\bar{y} = (y_1 + y_2 + \dots + y_n)/n. \quad (3.19)$$

The best line by this method is then the one of slope m passing through the point $(\bar{x}; \bar{y})$.

The differences $(y_j - y_i)$ provide six measurements of the quantity d_y and hence six measurements of m . They can be used for estimating the error as discussed in Chapter 1.

Example 3.4

Given the following table of values, calculate a value for the slope of the straight line through them by using the points-in-pairs method. Also determine the values of x and y for one point through which you would draw the graph,

x :	2.0	4.0	6.0	8.0	10.0	12.0
y :	7.1	8.2	8.8	10.0	11.0	11.8

y -differences:

$$y_4 - y_1 = 2.9$$

$$y_5 - y_2 = 2.8$$

$$y_6 - y_3 = 3.0$$

Average difference:

$$\bar{d}_y = (2.9 + 2.8 + 3.0)/3 = 2.90$$

Slope:

$$m = \bar{d}_y / \bar{d}_x = 2.9/6 = 0.48$$

x -differences:

$$\bar{d}_x = x_4 - x_1 = 6$$

$$= x_5 - x_2 = 6$$

$$= x_6 - x_3 = 6$$

Average x:

$$\begin{aligned}\bar{x} &= (2.0+4.0+6.0+8.0+10.0+12.0)/6 \\ &= 7.0\end{aligned}$$

Average y:

$$\begin{aligned}\bar{y} &= (7.1+8.2+8.8+10.0+11.0+11.8)/6 \\ &= 9.5\end{aligned}$$

Therefore the graph can be drawn through the point

$$(\bar{x}; \bar{y}) = (7.0; 9.5).$$

To calculate the error on m, values of m are calculated for each value of $y_j - y_i$. Three values are obtained, viz.

$$\begin{array}{lll}m_1 = (y_4 - y_1)/d_x & m_2 = (y_5 - y_2)/d_x & m_3 = (y_6 - y_3)/d_x \\ = 2.9/6 & = 2.8/6 & = 3.0/6 \\ = 0.48 & = 0.47 & = 0.50\end{array}$$

The mean value is

$$\begin{aligned}\bar{m} &= (0.48+0.47+0.50)/3 \\ &= 0.48.\end{aligned}$$

The adjusted standard deviation is

$$\begin{aligned}s &= \left[\sum_{i=1}^3 (m_i - \bar{m})^2 / (n-1) \right]^{\frac{1}{2}} \\ &= \left[\{(0.48-0.48)^2 + (0.47-0.48)^2 + (0.50-0.48)^2\} / (3-1) \right]^{\frac{1}{2}} \\ &= 0.02\end{aligned}$$

The final value for the slope is

$$\therefore m = 0.48 \pm 0.02$$

3.5.3 Linear Regression

If we know that the results of an experiment can be represented by a straight line, the values of m and c can be calculated also from statistical formulas, which are based on the principle of *least squares*. Suppose we have n pairs of values (x_1, y_1) , (x_2, y_2) , ..., (x_n, y_n) and the relation between them can be represented by equation (3.4), then it can be shown that the slope is given by

$$m = \left[n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i \right] / D \quad (3.20)$$

or written in abbreviated form as

$$m = [n \Sigma xy - \Sigma x \Sigma y] / D$$

and the y -intercept

$$c = \left[\Sigma x^2 \Sigma y - \Sigma x \Sigma xy \right] / D \quad (3.21)$$

where

$$D = n \Sigma x^2 - \left[\Sigma x \right]^2. \quad (3.22)$$

The errors in m and c are given by

$$s_m^2 = \left(\frac{n}{n-2} \right) \frac{\Sigma \delta_i^2}{D} \quad (3.23)$$

and

$$s_c^2 = \left[\frac{1}{n-2} \right] \frac{\Sigma \delta_i^2 \Sigma x^2}{D} \quad (3.24)$$

where δ_i is given by

$$\delta_i = y_i - mx_i - c. \quad (3.25)$$

The correlation coefficient is defined as

$$r = \frac{[n\sum xy - \sum x \sum y]}{[D\{n\sum y^2 - (\sum y)^2\}]^{\frac{1}{2}}}. \quad (3.26)$$

The value of r is zero if there is no correlation, i.e. the points are distributed at random over the graph. If all points are exactly on the straight line, $r = 1$.

Although only the final results have been listed above, you must be able to perform a linear regression analysis on your results. *Please note that this method must be applied in all assignment problems for this Chapter, unless otherwise stated.*

Example 3.5

We use the same data listed in example 3.4 viz.

x :	2.0	4.0	6.0	8.0	10.0	12.0
y :	7.1	8.2	8.8	10.0	11.0	11.8

Step 1 : We draw a table in which the required values are listed.

i	x_i	y_i	$x_i y_i$	x_i^2	$mx_i + c$	δ_i^2
1	2	7.1	14.2	4	7.1	0.00
2	4	8.2	32.8	16	8.1	0.01
3	6	8.8	52.8	36	9.0	0.04
4	8	10.0	80.0	64	9.9	0.01
5	10	11.0	110.0	100	10.9	0.01
6	12	11.8	141.6	144	11.8	0.00
$\sum_{i=1}^6$	42	56.9	431.4	364		0.07

Step 2: Before the last two columns can be completed, we must calculate m and c . From the table and equations (3.22), (3.20) and (3.21) it follows that

$$\begin{aligned} D &= (6 \times 364) - (42)^2 \\ &= 420, \\ m &= [6 \times 431.4 - 42 \times 56.9]/420 \\ &= 0.47 \end{aligned}$$

and

$$\begin{aligned} c &= [364 \times 56.9 - 42 \times 431.4]/420 \\ &= 6.17 \end{aligned}$$

Step 3: Columns 6 and 7 are now completed. Firstly, calculate $mx_i + c = 0.47x_i + 6.17$ for each x_i value, then subtract the corresponding y_i value and square the answer. These values are listed in column 7. For example, for $i = 4$ we obtain

$$\begin{aligned} mx_4 + c &= (0.47 \times 8) + 6.17 \\ &= 9.9. \end{aligned}$$

$$\begin{aligned} \therefore y_4 - (mx_4 + c) &= 10.0 - 9.9 \\ &= 0.1 \end{aligned}$$

$$\begin{aligned} \therefore \delta_4 &= (0.1)^2 \\ &= 0.01 \end{aligned}$$

The calculations for $i = 1, 2, 3, 5$ and 6 are similar.

Step 4: The errors follow from equations (3.23) and (3.24)

$$\begin{aligned} s_m &= \left[\frac{6}{(6-2)} \times \frac{0.07}{420} \right]^{\frac{1}{2}} \\ &= 0.02 \end{aligned}$$

and

$$s_c = \left[\frac{1}{(6-2)} \times \frac{(0.07)(364)}{420} \right]^{\frac{1}{2}}$$

$$= 0.12 .$$

The *best* straight line through the experimental points can therefore be written as

$$y = mx + c$$

$$\therefore y = (0.47 \pm 0.02)x + (6.17 \pm 0.12).$$

This *best* line is shown in figure 3.6. It is drawn by marking the y-intercept $c = (0; 6.17)$ on the y-axis. Select a convenient value of the x-axis, say $x = 10$, and *calculate* the corresponding y-value using the above equation, namely $y = 0.47x + 6.17 = 0.47(10) + 6.17 = 10.87$.

Mark this point $(10; 10.87)$ on the graph and draw the *best* line between this point and $(0; 6.17)$.

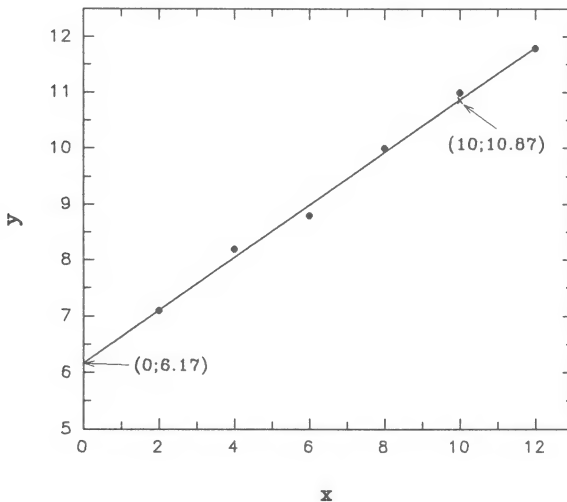


Figure 3.6 Graph of y as a function of x for example 3.5.

3.6 Hints on how to plot graphs

The graphical presentation of data is a very important aspect of the PHY103 module. Each assignment requires the plotting of a number of graphs and each experiment that will be executed during the laboratory course includes amongst other things the plotting of at least one graph. If you pay careful attention to the hints listed below, you will most probably not be penalised for imperfect graphs. However, if you don't pay attention to these hints, you will most certainly lose a lot of marks.

We advise you to keep these hints at hand and to consult them while you are plotting a graph. Make sure that your graph complies with all the requirements before you proceed to a next task. The hints are the following :

1. Usually one quantity (the *independent variable*) is changed by given amounts and the effect on the other quantity (the *dependent variable*) is measured. The independent variable is plotted along the horizontal axis (the *abscissa*) and the dependent variable along the vertical axis (the *ordinate*).
2. Choose scales such that the experimental points are as *widely spread* as possible over the sheet of paper.
3. Choose *simple scale divisions*. This will reduce the labour involved in estimating to fractions of a division and will also reduce the risk of making a mistake.
4. Always *label the axis* with the name of the quantity being plotted along that axis and also the *units* in which the quantity is measured.
5. Give the graph a *heading* by stating which quantity is plotted as a function of what other quantity.

6. When taking readings, try to *spread them out evenly* over the range of values of the quantity being measured.
7. If possible, *plot a graph as your experiment proceeds*. You can check immediately if a point is so widely off as to need repeating. You will see immediately whether the distribution of points is in accordance with the considerations outlined under point 6.
8. Indicate datapoints *accurately and clearly* by a cross "x" or a dot "•", don't use a blob.
9. If errors on individual datapoints are known, indicate Δx values by $|\bullet|$ according to scale and Δy values by $\overline{|}$ and if both Δx and Δy are known by $|\overline{|}$ (look at the graph in chapter 4).

Example 3.6

A student was asked to investigate how the time of free fall of a steel sphere depends on the distance fallen and to deduce a value for g , the acceleration due to gravity. The apparatus used enables the time of the fall of the sphere to be measured directly with an electric timer. The distance h can be varied over a range 0 to 1.5 m and the timer is accurate to 0.005 s. The student obtained the results listed below.

Distance h (m)	Time t (s)
0.20	0.200
0.40	0.285
0.60	0.350
0.80	0.400
1.00	0.450
1.20	0.495
1.40	0.535
1.50	0.550

According to theory distance h is related to time t by

$$h = \frac{1}{2}gt^2.$$

The student plotted the graph of h against t as shown in figure 3.7.

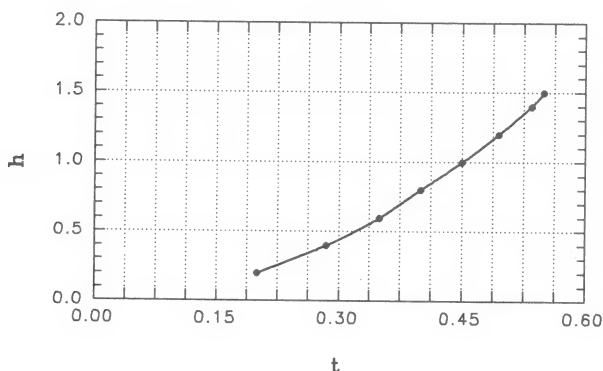


Figure 3.7 Graph of h as a function of t .

The student plotted the incorrect graph to verify that the data satisfy the equation $h = \frac{1}{2}gt^2$. He should have plotted t^2 against h to give a straight line graph of slope $2/g$ (note that h is the independent variable. Since the value of h can be set at convenient values and the time t then measured as a function of h). Plotting h against t gives a parabola from which no useful information can be obtained easily.

Other bad features in this graph are:

1. 0.1 s on the t axis is represented by 2.67 divisions, making accurate plotting of points very difficult. A better choice would have been 0.1 s represented by 2 divisions.
2. The scale along the h axis could easily be doubled.
3. Units of h and t are not given.
4. The graph has no title.

The first step in plotting the correct graph is to set up a table listing the values of h and t^2 .

h (m)	t^2 (s ²)
0.20	0.040
0.40	0.081
0.60	0.123
0.80	0.160
1.00	0.203
1.20	0.245
1.40	0.286
1.50	0.303

A graph of t^2 against h is drawn as shown in figure 3.8.

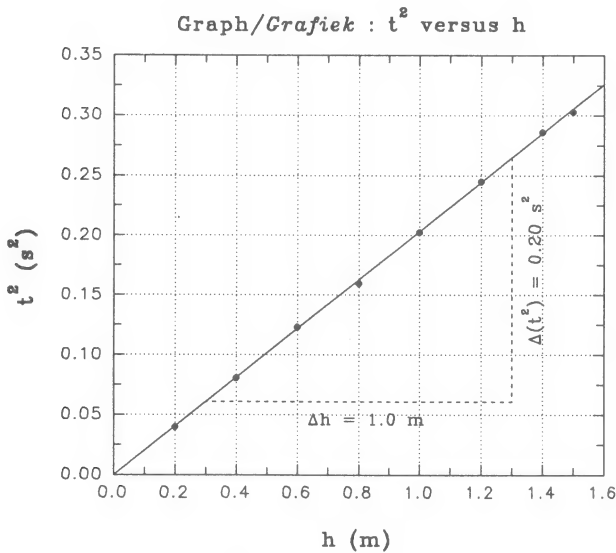


Figure 3.8 Graph of t^2 as a function of h .

From the graph it follows that

$$m = \frac{\Delta(t^2)}{\Delta h} = \frac{0.20 \text{ s}^2}{1.00 \text{ m}} = 0.20 \text{ s}^2 \text{ m}^{-1}$$

$$m = \frac{2}{g} \Rightarrow g = \frac{2}{m} = \frac{2}{0.20 \text{ s}^2 \text{ m}^{-1}} = 10 \text{ m s}^{-2}$$

It is suggested that you use the linear regression method to calculate m , c , s_m and s_c .

3.7 Questions

- 3.1 A student was supplied with a stop watch, two metre rules and a simple pendulum suspended from a ceiling and was asked to measure the height of the ceiling indirectly. He set the pendulum swinging through a small angle and measured the period of oscillation for different lengths of the pendulum. Since he was unable to measure the length of the pendulum directly, he measured the height of the centre of the pendulum bob above the floor. He obtained the results tabulated below.

Height of bob above floor (mm)	Time for 50 oscillations (s)
400	155.3
600	148.8
800	142.2
1000	134.0
1200	127.4
1400	119.2
1600	110.5

The period T of the pendulum of length ℓ is given by

$$T = 2\pi\sqrt{\ell/g}$$

where g is the acceleration due to gravity. But $\ell = H - h$ where H is the height of the ceiling and h is the height of the centre of the pendulum above the floor. Therefore

$$T = 2\pi[(H-h)/g]^{\frac{1}{2}}.$$

Plot a suitable graph to find the height of the ceiling H from the two intercepts. Assume that $g = 9.8 \text{ m s}^{-2}$ (g could be obtained from the slope of the graph). Having obtained your values for H , answer the following questions :

- a) Which value of H do you consider the least accurate?
Give reasons for your choice and explain how the accuracy could have been improved.
- b) Why was the bob set swinging through a small angle?
- c) Why was h measured to the centre of the bob?
- d) Why was the completed number of oscillations chosen to be large?
- e) Can you see any advantage in measuring the height of the ceiling in this way?

- 3.2 A light wooden uniform metre rule was clamped to the surface of a bench so that a length L of the metre rule was overhanging the edge. The end of the latch was loaded with a mass and the depression D produced at the loaded end was measured. Values of D were obtained for different lengths L .

Depression D (mm)	Length L (mm)
2.0	200
6.0	300
13.0	400
28.0	500
49.0	600
75.0	700
109.0	800

Assume that $D = KL^n$, where K and n are constants, draw a suitable graph to determine n and K . Use the graph to find the value of L for $D = 0.5$ mm.

- 3.3 In a certain experiment the total pressure exerted on a gas was the pressure P , recorded on a gauge, plus the unknown atmospheric pressure B . Corresponding values of P and the volume V were recorded. On the assumption that the total pressure was inversely proportional to V , P was plotted against $1/V$. The points-in-pairs approach gave the slope to be 1.10×10^9 Nm, the mean values of P and $1/V$ were respectively 215×10^3 Nm⁻² and 2.85×10^{-4} m⁻³. The value of B by estimating the value of an intercept.

- 3.4 Fit a linear relation to the following data:

x :	10	12	13	17	19	20
y :	11.0	7.6	6.2	-0.1	-3.2	-5.0

- 3.5 Values of the surface tension of water, γ Nm⁻¹, at different temperatures, T °C, are given below. If $\gamma = a - b\theta$, where θ is the temperature on the kelvin scale, find the most probable values of a and b .

T :	10	20	30	40	50	60
γ :	0.074	0.073	0.071	0.070	0.068	0.066

- 3.6 Values of the surface tension of bromobenzene, $\gamma \text{ Nm}^{-1}$, at different temperatures, T °C, are given below. Fit a law of the form $\gamma = \gamma_0[1 - (\theta/670)]^n$ where θ is the temperature in degrees kelvin.

T :	15	42	49	78	90	105	125
γ :	0.038	0.035	0.034	0.031	0.029	0.028	0.025

- 3.7 Fit a linear law to the values of x and y given below:

x :	0	1	2	3	4	5	6	7	8	9
y :	4.6	7.1	9.5	11.5	13.7	15.9	18.6	20.9	23.5	25.4

Plot the values of x and y and draw the straight line that fits the points best.

- 3.8 Find the resistance R_0 at 0 °C and the temperature coefficient α , where the resistance $R = R_0(1 + \alpha T)$ for a sample of wire giving the following data:

R (ohm) :	2.0700	2.1465	2.2225	2.2990	2.3740	2.4515
T (°C) :	10	20	30	40	50	60

- 3.9 Observations are made of the excess temperature θ of a cooling body as a function of time t. Determine whether the data follows Newton's law of cooling, $\theta = \theta_0 e^{-bt}$ and if so, determine the constants θ_0 and b by graphical methods.

t (min) :	0	4	8	12	16	20
θ (°C) :	40.5	27.0	18.0	12.0	8.0	5.3

- 3.10 Find the constants A and B for Cauchy's equation ($\mu = A + B/\lambda^2$) for the index of refraction of light through a refracting medium. The given data is for sodium light refracted through water.

Does the equation appear to be inaccurate in any region? If so, state which region.

μ	:	1.4040	1.3435	1.3371	1.3330	1.3307	1.3289
λ (Å):		2144	3968	4861	5893	6708	7685

Chapter 4

Planning an experiment and writing a report

Science is built up with facts as a house with stones; but a collection of facts is no more a science than a heap of stones is a house.

Please no exploded trolleys!

Jules Poincaré (1854–1912)

4.1	Introduction	94
4.2	Recording measurements	94
4.3	Writing a report	95
4.4	A model report	97

4.1 Introduction

The time available for performing an experiment is nearly always limited. During the laboratory course you will be expected to complete one experiment per day. If the time required for other activities such as lectures, tests, demonstrations and the processing of experimental data is taken into account, only a few hours are left for the execution of an experiment. You will find it absolutely essential to use time efficiently and to *plan* every experiment before you attend the laboratory course. If you don't follow this advice, you might find that you don't have adequate time to plan and perform the experiment during the limited time available.

In every experiment it is essential to keep a running record of everything that is done. In this chapter some suggestions for keeping the record are given. The important thing is not that you should regard them as a set of rules to be followed blindly, but rather that you should understand the spirit behind them, which is to produce a record – accurate, complete and clear – with the minimum of effort.

4.2 Recording measurements

All measurements should be recorded *immediately and directly in ink* in the examination book which will be supplied by the University. There is no exception to this rule. As soon as you have taken all your measurements, one of the laboratory instructors will sign all the pages on which you recorded your results. If these pages are not included in your final report, you will be penalised.

All special observations and suggestions should be written down in this book. Preliminary graphs to see whether the distribution of points is satisfactory, to calculate slopes and intercepts and to obtain the errors associated with these values, are drawn by a microcomputer (see Appendix E). All final graphs have to be drawn on graph paper supplied by the University.

4.3 Writing of a report

The communication of ideas, theories and experimental results is an important part of scientific work. We are going to consider some elementary features of good scientific writing in this section.

A report on the experiment should be divided into the following sections:

1. Introduction
2. Experimental method
3. Results
4. Discussion and conclusions

Although the actual plan of a report depends to some extent on its contents, it is obvious that the one suggested above is logical and you should try to follow it, at least in a general way. We consider each section in turn.

1. Introduction

The purpose of the introduction is to set the scene in general terms. In it you describe the aim of the experiment and give a *little* theoretical background to the problem. If the aim of the experiment

is to verify a formula, it should be *quoted* and *not derived*. You should avoid all details and technicalities in the introduction.

2. **Experimental method**

One picture is worth a thousand words. The importance of *diagrams* in a report can hardly be exaggerated. Combined with a few words of explanation, a diagram is often the easiest and most effective way of explaining the principle of the experiment, describing the apparatus and introducing notation.

3. **Results**

When possible, always record measurements in *tabular* form. This is both compact and easy to follow. Measurements of the same quantity should preferably be recorded vertically. Head each column with the name and/or symbol of the quantity, followed by the units. The results should give a faithful picture of the quality, accuracy and reproducibility of the measurements. All calculations and graphs required, should be shown here.

4. **Discussions and conclusions**

The discussion should include a comparison with the relevant theory. Your *conclusions*, the counterpart to the *object* stated in the introduction, should also be given. This is also the appropriate section to mention all special observations and recommendations on how to improve the experiment.

The final report must be written in an examination book.

4.4 A model report

As an example how a model report should be written, consider the following experiment:

The volume V of a cylinder is given by

$$V = \pi r^2 h \quad (4.1)$$

where r is the radius and h the height of the cylinder.

Six cylinders of the *same* length and made of the same metal, a vernier calliper as well as a measuring cylinder with water, are supplied. You are requested to determine the value of π experimentally.

During the experiment the diameter and the length of each cylinder are measured and the volume of each one is measured by lowering it into the measuring cylinder until it is totally submerged. The volume of water displaced by the metal is determined.

Here follows the report on the experiment:

REPORT: DETERMINATION OF THE VALUE OF π

1. Introduction

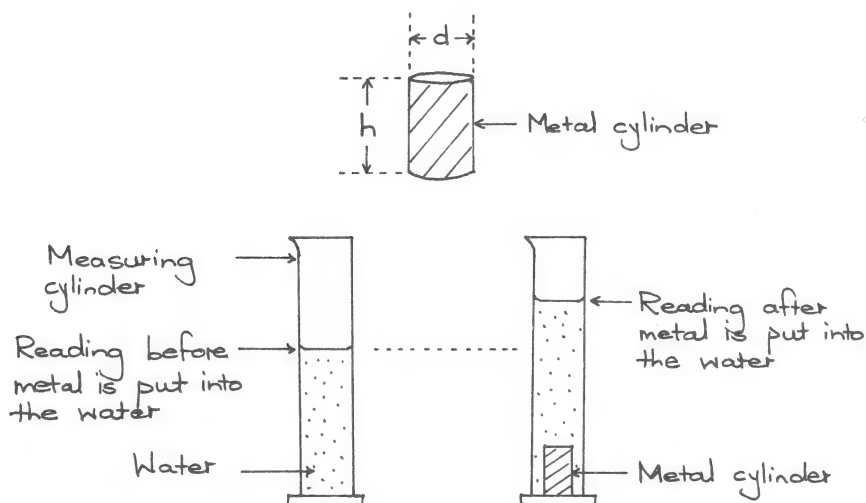
- 1.1 **Aim:** To determine the value of π by using metal cylinders and to compare the value obtained with the known value of $\pi = 3.141593\dots$

1.2 **Theory:** The volume V of a cylinder is given by

$$V = \pi r^2 h \quad (1)$$

where r is the radius and h the height of the cylinder.

2. Experimental method



- 2.1 The diameter d of each cylinder was measured five times with vernier callipers.
- 2.2 The length h of each cylinder was measured once.
- 2.3 Each cylinder was put three times into the measuring cylinder and the volume of the displaced water determined.

3. Results

3.1 Tables

Table 1 : Measurements

Cylinder no.	Height h(mm)	Diameter d(mm)					Volume of water (cm ³)					
		d ₁	d ₂	d ₃	d ₄	d ₅	B ₁	A ₁	B ₂	A ₂	B ₃	A ₃
1	80.2	10.1	10.2	10.0	10.1	10.0	67	73	70	76	76	81
2	80.1	15.1	15.0	14.9	14.9	15.1	62	77	68	82	72	85
3	79.8	17.6	17.3	17.5	17.4	17.2	57	76	62	82	68	87
4	80.2	20.3	20.1	20.1	19.9	20.1	53	78	57	83	61	86
5	79.9	21.9	22.1	22.2	22.0	21.8	48	79	46	75	55	85
6	79.8	24.9	25.0	25.1	24.9	25.1	45	85	49	88	51	92

B = Before metal is put into the water

A = After metal is put into the water

Table 2 : Average values of measured quantities

Cylinder no.	Average diameter d (10 ⁻² m)	Radius r (10 ⁻³ m)	r ² (10 ⁻⁵ m ²)	Average Volume V̄ (10 ⁻⁵ m ³)
1	1.01±0.01	5.05±0.05	2.50±0.05	0.6±0.1
2	1.50±0.01	7.50±0.05	5.63±0.08	1.4±0.1
3	1.74±0.02	8.70±0.10	7.57±0.17	1.9±0.1
4	2.01±0.01	10.05±0.05	10.10±0.10	2.5±0.1
5	2.20±0.02	11.00±0.10	12.10±0.22	3.0±0.1
6	2.50±0.01	12.50±0.05	15.63±0.12	4.0±0.1

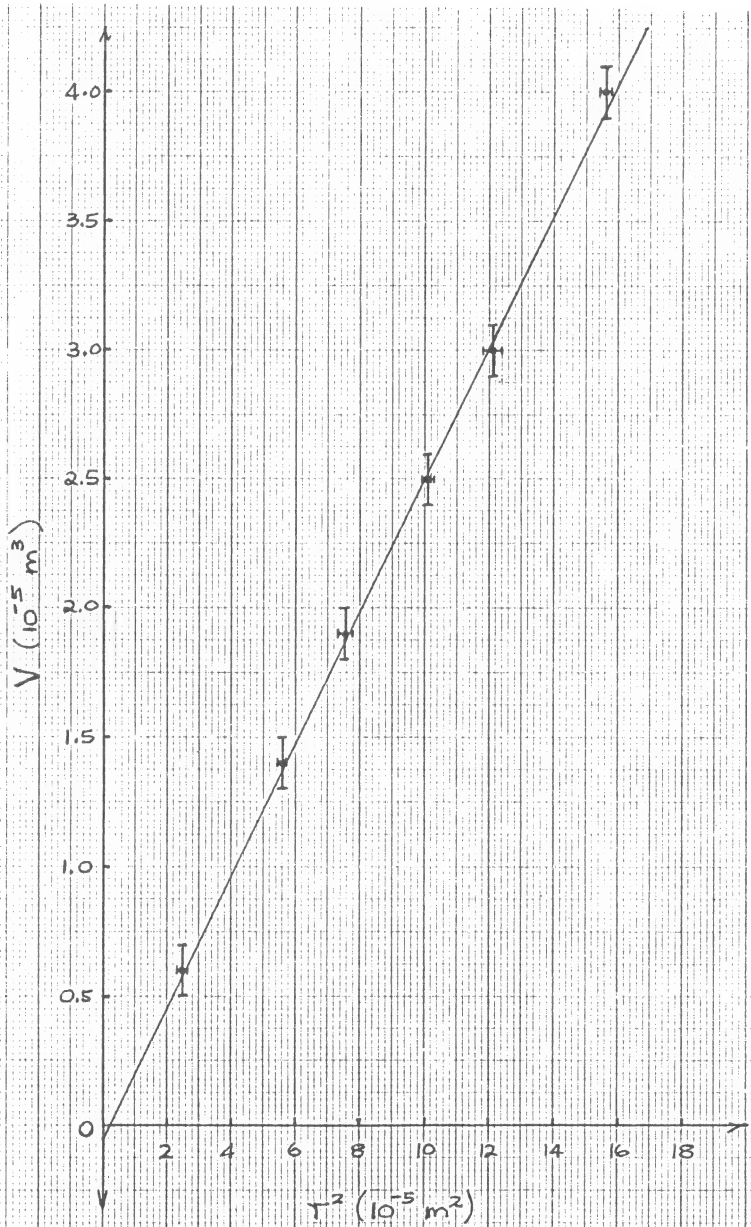
Average height h = 80.0±0.2 mm = (8.00±0.02) × 10⁻² m.

(In this table $V_1 = B_1 - A_1$, etc. The average volume is $\bar{V} = (V_1 + V_2 + V_3)/3$).

(Note that the errors on h and in columns 2 and 5 are adjusted standard deviations and the errors in columns 3 and 4 were obtained with the equations in § 2.5.9).

3.2 Graph

From equation (1): $V = (\pi h)r^2$



which has the form of a straight line

$$y = mx + c$$

with

$$y = V ; x = r^2 ; m = \pi h ; c = 0.$$

3.3 Calculations

The best straight line through the points was obtained by means of a linear regression analysis. The necessary summations are listed in Table 3.

Table 3 : Results of linear regression analysis.

i	x_i (10^{-5}m^2)	y_i (10^{-6}m^3)	$x_i y_i$ (10^{-10}m^5)	x_i^2 (10^{-10}m^4)	$mx_i + c$ (10^{-6}m^3)	δ_i^2 (10^{-14}m^6)
1	2.50	6	1.50	6.25	5.88	1.44
2	5.63	14	7.88	31.70	13.89	1.21
3	7.57	19	14.38	57.30	18.86	1.96
4	10.10	25	25.25	102.01	25.34	11.56
5	12.10	30	36.30	146.41	30.46	21.16
6	15.63	40	62.52	244.30	39.49	26.01
Σ	53.53	134	147.83	587.97		63.34

$$D = n \Sigma x^2 - (\Sigma x)^2$$

$$= 6(587.97 \times 10^{-10} \text{m}^4) - (53.53 \times 10^{-5} \text{m}^2)^2$$

$$= 6.624 \times 10^{-8} \text{m}^4.$$

Slope :

$$\begin{aligned}
 m &= \frac{[n\Sigma xy - \Sigma x\Sigma y]}{D} \\
 &= \frac{6(147.83 \times 10^{-10} \text{ m}^5) - (53.53 \times 10^{-5} \text{ m}^2)(134 \times 10^{-6} \text{ m}^3)}{6.624 \times 10^{-8} \text{ m}^4} \\
 &= 2.56 \times 10^{-1} \text{ m}.
 \end{aligned}$$

y-intercept :

$$\begin{aligned}
 c &= \frac{[\Sigma x^2 \Sigma y - \Sigma x \Sigma xy]}{D} \\
 &= \frac{(587.97 \times 10^{-10} \text{ m}^4)(134 \times 10^{-6} \text{ m}^3) - (53.53 \times 10^{-5} \text{ m}^2)(147.83 \times 10^{-10} \text{ m}^5)}{6.624 \times 10^{-8} \text{ m}^4} \\
 &= -5.21 \times 10^{-7} \text{ m}^3.
 \end{aligned}$$

Errors :

$$\begin{aligned}
 s_c^2 &= \frac{\Sigma \delta^2 \Sigma x^2}{(n-2)D} = \frac{(63.34 \times 10^{-14} \text{ m}^6)(587.97 \times 10^{-10} \text{ m}^4)}{(4)(6.624 \times 10^{-8} \text{ m}^4)} \\
 &= 1.406 \times 10^{-13} \text{ m}^6
 \end{aligned}$$

$$\therefore s_c = 3.75 \times 10^{-7} \text{ m}^3.$$

$$s_m^2 = \frac{n\Sigma \delta^2}{(n-2)D} = \frac{6(63.34 \times 10^{-14} \text{ m}^6)}{4(6.624 \times 10^{-8} \text{ m}^4)}$$

$$= 1.434 \times 10^{-5} \text{ m}^2$$

$$\therefore s_m = 3.79 \times 10^{-3} \text{ m}.$$

The y axis intercept is: $c = (-5.21 \pm 3.75) \times 10^{-7} \text{ m}^3$

$$= (-0.5 \pm 0.4) \times 10^{-6} \text{ m}^3.$$

The slope is: $m = (2.56 \times 10^{-1}) \pm (3.79 \times 10^{-3}) \text{ m}$

$$= (2.56 \pm 0.04) \times 10^{-1} \text{ m}.$$

The value of π is:

$$\pi = \frac{m}{h} = \frac{2.56 \times 10^{-1} \text{ m}}{8.00 \times 10^{-2} \text{ m}} = 3.20.$$

The standard error on π follows form:

$$\left(\frac{\Delta \pi}{\pi}\right)^2 = \left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta h}{h}\right)^2$$

$$\therefore \Delta \pi = (3.20) \left[\left(\frac{0.04 \times 10^{-1} \text{ m}}{2.56 \times 10^{-1} \text{ m}} \right)^2 + \left(\frac{0.02 \times 10^{-2} \text{ m}}{8.00 \times 10^{-2} \text{ m}} \right)^2 \right]^{\frac{1}{2}}$$

$$= 0.05.$$

Thus:

$$\pi = 3.20 \pm 0.05.$$

$$= 3.2 \pm 0.1.$$

(The value of π is rounded off to *two* significant digits, since the volume in Table 1 could only be determined accurately to *two* significant digits).

4. Discussion and Conclusion

The known value of $\pi = 3.141593 \dots$ lies within the experimental range of (3.1 - 3.3).

Chapter 5

Laboratory apparatus

For we do not think that we know a thing until we are acquainted with its primary conditions or first principles.

Aristotle (384–322 BC)

5.1	Introduction	106
5.2	Meter stick	106
5.3	Vernier scale	106
5.4	Electric meters	111
	5.4.1 Galvanometer	111
	5.4.2 Ammeter	113
	5.4.3 Voltmeter	115
	5.4.4 Digital multimeters	117
5.5	Power supplies	119
5.6	Cathode ray oscilloscope (CRO)	120
5.7	Questions	123

5.1 Introduction

In this chapter the characteristics and operation of a few simple laboratory apparatus that are widely used, will be discussed. Not all the details of any instrument will be given and only some of the instruments that you will be using during the practical course are included. Special instruments such as the e/m apparatus, linear air track, etc. will be discussed in detail in later chapters along with the experiments requiring their use.

5.2 Meter stick

The wooden meter stick is usually used to measure length when the accuracy desired is not very high. A metre stick can be read with an accuracy of about 0.5 mm and can therefore not be used to measure very small lengths or where very high accuracy is required. In these cases the micrometer or the vernier callipers are used. These instruments have *vernier scales* to improve the accuracy of the measurement. If the principle behind a vernier scale is understood, any of these instruments can be used without any difficulty.

5.3 Vernier scale

A vernier consists of a scale which can move next to a stationary scale. It has divisions which are slightly smaller than a division on the stationary main scale along which it slides. If the zero mark on the vernier scale is on any main scale division, n divisions of the vernier scale correspond to $n-1$

divisions on the main scale. Therefore each vernier division is $(n-1)/n = 1 - 1/n$ of the main scale divisions. Consequently the vernier division is $1/n$ shorter than the main scale division. The quantity $1/n$ of the main scale division is called the "least count" of the vernier.

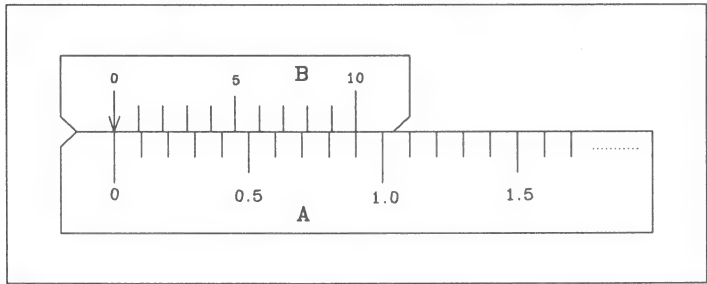


Figure 5.1 Illustration of a vernier scale in the zero position.

In figure 5.1, A and B are the main scale and the vernier scale respectively. B can move next to A. The divisions on the vernier are from 0 to 10 and has a total length equal to 9 divisions on the main scale. Therefore a vernier division is equal to $1 - 1/10 = 0.9$ of a division on the main scale. If B is moved until the first division (the first mark to the right of 0) coincides with the 0.1 mark on the main scale, then the distance between the 0 on the main scale and the 0 on the vernier scale is $1/10$ of a scale division on the main scale. Similarly, if the second mark to the right of 0 on B coincides

with the 0.2 mark on the main scale, the distance between the two zeros is equal to $2/10$ of a scale division on the main scale. The vernier enables us therefore to measure to $1/10$ of a scale division on the main scale by simply observing which mark on the vernier scale coincides with which mark on the main scale.

Another example of a vernier scale is shown in figure 5.2. Try to read the correct value before you proceed with the rest of the discussion.

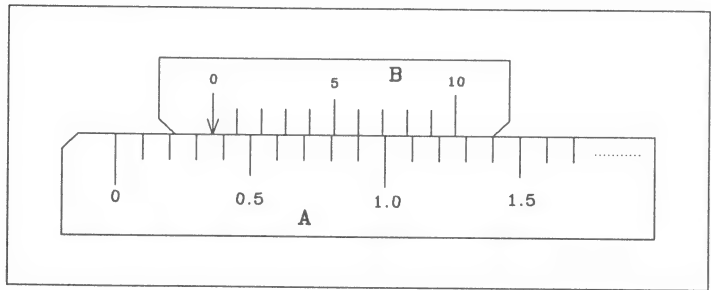


Figure 5.2 Illustration of a vernier scale in an off-zero position.

The arrow on the vernier scale and the fourth mark on the main scale coincide. The reading is therefore between 0.3 and 0.4. The second decimal figure is obtained from the vernier scale. The sixth mark on this scale coincides with a mark on the main scale. Therefore the second number is 6 and the complete reading is 0.36.

An angular vernier is used in many optical instruments. A typical angular vernier is shown in figure 5.3. In this case 29 main scale divisions (upper scale) cover 30 divisions on the vernier scale. The smallest division on the main scale is 0.5° and the vernier scale enables us to measure to an accuracy of $0.5/30 = 1/60$ of a degree (i.e. 1 second). The reading on the top photograph is $(12.5 + 0.5 \times 15/30)^\circ = 12.75^\circ$. How have we arrived at this answer? The zero mark on the vernier scale is between 12.5° and 13.0° . Therefore we have to add something to 12.5° (the first term between brackets). The 15th mark on the vernier scale coincides with a mark on the main scale. The angle that has to be added is therefore $15/30$ of a division on the main scale. Since a division of the main scale is 0.5° , the additional angle is $0.5 \times 15/30^\circ$.

Now you can try to read the angle on the photograph in figure 5.3. The answer is 192.73° .

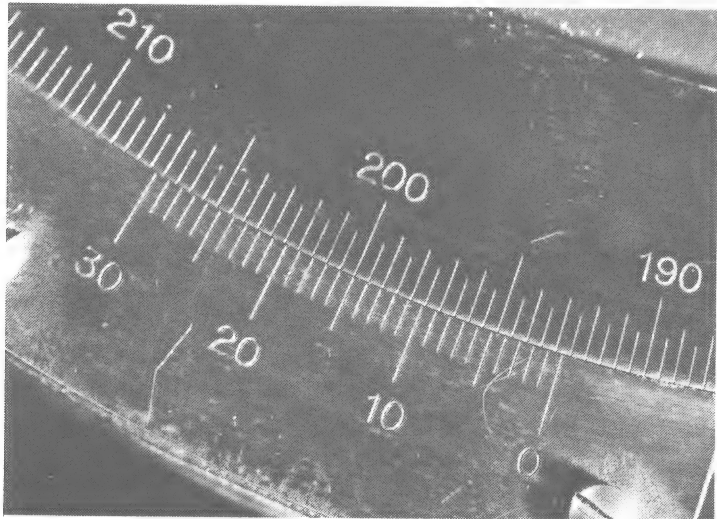
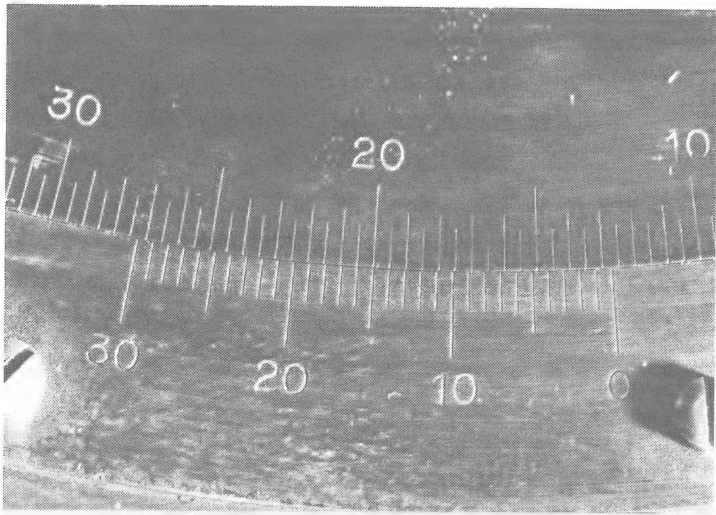


Figure 5.3 Angular vernier.

5.4 Electric meters

5.4.1 Galvanometer

The moving-coil galvanometer consists of a flat coil supported in the magnetic field of a permanent magnet as shown in figure 5.4. If electrons move through the coil, they experience a magnetic force which causes the coil to turn against the resisting torque of the spring s . It can be shown (see Problem 5.10) that the torque on the coil due to the current I flowing through it is given by

$$\Gamma = NIAB\sin\theta, \quad (5.1)$$

where A is the area of the coil, N is the number of turns on the coil, B is the magnitude of the magnetic field and θ is the angle between the normal to the plane of the coil and the vector $\vec{B}$. However, due to the design of the magnet, $\vec{B}$ is always perpendicular to the plane of the coil, i.e. $\theta = 90^\circ$. Equation (5.1) therefore reduces to

$$\Gamma = NIAB. \quad (5.2)$$

As soon as the coil starts to turn, the spring s winds up and gives rise to a restoring torque. According to Hooke's law this restoring torque is proportional to the angle through which the coil is rotated, i.e.

$$\tau = k\theta \quad (5.3)$$

where k is the torsional constant of the spring. The equilibrium position of the coil is reached when

$$\Gamma = \tau$$

or

$$NIAB = k\theta.$$

Therefore

$$\theta = NIAB/k. \quad (5.4)$$

As shown in figure 5.4, a pointer P is attached to the axis of the galvanometer and indicates the rotation angle θ on a scale. This scale may be calculated against known currents. In this case the galvanometer can be used as an ammeter.

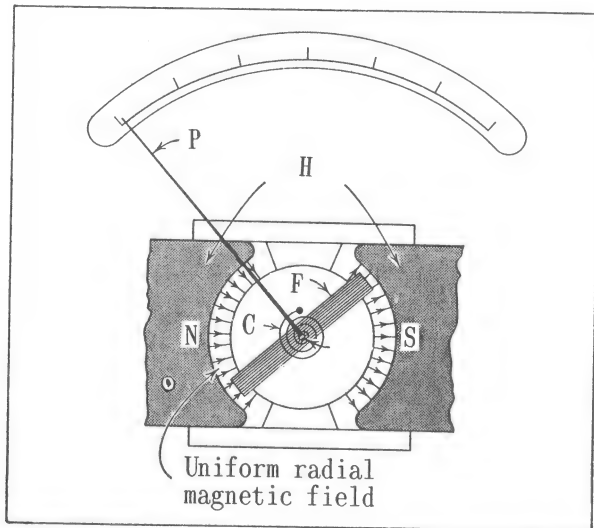


Figure 5.4 Galvanometer.

5.4.2 Ammeter

When a galvanometer is used as a current-measuring instrument (*ammeter*), it is desirable that its *resistance is very low* compared with other elements in the circuit to which it is connected. This is necessary to avoid perturbing the circuit. A simple example is shown in figure 5.5 where the idea is to determine the resistance R by measuring the current flowing through it when it is connected to a known voltage source V . We should like to have the resistance of the galvanometer much less than R so that all but a negligible fraction of the voltage V appears across R . Galvanometers for use in ammeters are wound with a few turns of heavy wire to keep the resistance low.

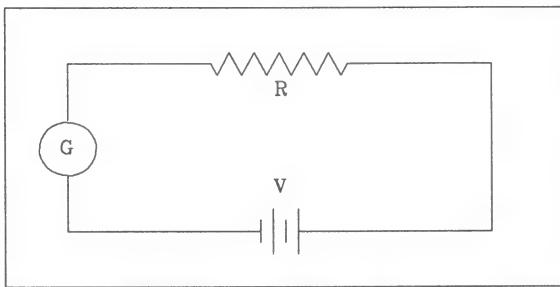


Figure 5.5 Galvanometer G connected in series with resistor R .

The maximum sensitivity of an ammeter is fixed by the sensitivity of the galvanometer of which it is made. However, it is often required to

reduce the sensitivity to a lower value. This is done by providing a *shunt* or parallel resistance as shown in figure 5.6. The resistance of the galvanometer is given by R_g and the shunt resistance is R_s . Application of Ohm's law to this circuit gives the value of R_s required to reduce the sensitivity to any desired value. Such a shunt also reduces the total resistance of the ammeter.

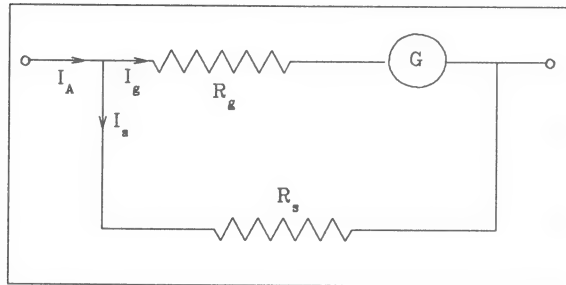


Figure 5.6 Galvanometer used as an ammeter.

Example 5.1

A $100\ \Omega$ galvanometer gives a full scale deflection for a current of 10^{-6} A. Determine the resistance of the shunt that will convert the galvanometer to an ammeter with a full scale deflection for 10^{-2} A.

The current through the shunt is given by

$$I_s = I_A - I_g.$$

Since the galvanometer is in parallel with the shunt, the potential difference across both is the same.

Therefore

$$I_s R_s = I_g R_g$$

or

$$(I_A - I_g) R_s = I_g R_g$$

and

$$\begin{aligned} R_s &= I_g R_g / (I_A - I_g) \\ &= \frac{10^{-6} [A] \times 100 [\Omega]}{(10^{-2} - 10^{-6}) [A]} \\ &= 10^{-2} \Omega. \end{aligned}$$

5.4.3 Voltmeter

When a galvanometer is to be used as a *voltmeter*, as in figure 5.7, it is necessary that the *resistance* be *much larger* than any resistance across which it is to be connected. Otherwise the voltmeter will alter the current flow in the circuit. Galvanometers to be used as voltmeters are wound with many turns of fine wire to give high sensitivity and high resistance. Usually there is sensitivity to spare and the total resistance of the voltmeter is increased and the

sensitivity is reduced to the desired value by the addition of the appropriate series resistance R_s as shown in Figure 5.8. A simple calculation again allows the determination of the appropriate value of R_s for a given sensitivity. Further discussions on the construction of volt- and ammeters are given in Chapter 9.

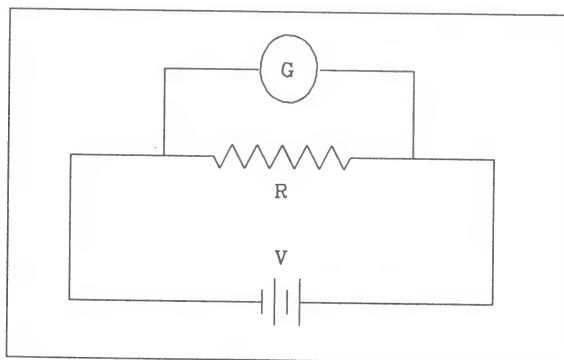


Figure 5.7 Galvanometer connected parallel with resistor R .

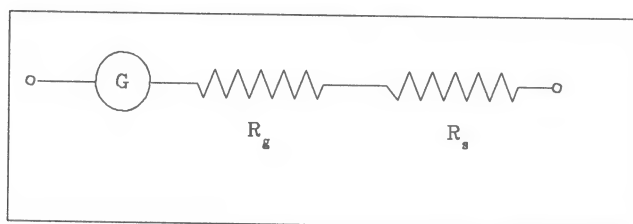


Figure 5.8 Galvanometer used as a voltmeter.

Example 5.2

Determine the resistance R_s required in figure 5.8 to give the voltmeter a full scale deflection for a potential difference of 100 V. $R_g = 100 \, \Omega$ and the galvanometer has a full scale deflection for $I_g = 10^{-6} \, \text{A}$.

Application of Ohm's law yields

$$I_g = V_{ab} / (R_g + R_s)$$

$$\therefore R_g + R_s = V_{ab} / I_g$$

$$\therefore R_s = V_{ab} / I_g - R_g$$

$$= \frac{100[\text{V}]}{10^{-6}[\text{A}]} - 100[\Omega]$$

$$= (10^4 - 100) \, \Omega$$

$$= 9900 \, \Omega$$

5.4.4 Digital multimeters

The digital multimeter is a laboratory instrument used to measure electrical potential differences, resistances and currents, and has in recent years almost totally replaced the more traditional multimeters, such as the familiar "AVO". The reason for the domination of the digital multimeter over the traditional meters is the "recent" development of low cost high quality liquid crystal displays,

analog to digital converts, etc. These developments enable electronic meters to be produced which are more accurate and cheaper to manufacture than the traditional analog electromechanical meter.

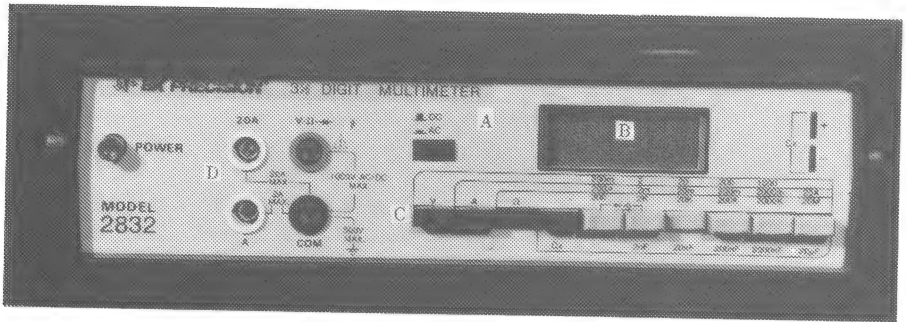


Figure 5.9 Digital multimeter.

The digital multimeter shown in figure 5.9, is a typical high quality general purpose digital multimeter. A digital multimeter externally consists of four main parts:

1. The case A.
2. The display B where the value of the measured quantity is displayed.
3. The range/function selector knobs C, which enables the required function, etc. such as volts to be selected.

4. The input terminals D. These terminals are used in conjunction with the range/function selector to select either potential difference, resistance or currents.

The use of the digital multimeter is very simple and is summed up in the following rules:

1. Switch the digital multimeter on by selecting the required range/function.
2. Wait for the internal self test to be performed. Only use the instrument if it passes the test.
3. Plug the test probes E into the correct input terminals as selected by the range/function switch.
4. Connect the test probes to the circuit under test. Where possible try to estimate if the currents/potential differences present in the circuit exceed the measuring range of the digital multimeter. *If they do, do not perform the measurement.*
5. Wait for the display to stabilise and then read the display.

5.5 Power supplies

Power supplies are divided into two groups, direct current (d.c) supplies and alternating current (a.c.) supplies. In our laboratory all supplies are solid state d.c. power supplies. Each power supply meets the requirements of the specific experiment in which it is used. A typical power supply is shown in figure 5.10. The instrument is connected to a 220 V wall outlet and put into operation by means of the ON/OFF switch. Both the current and the voltage of the output are variable by means of controls. It is also equipped with a meter which could be used as either a volt- or an ammeter.

You are advised to use this meter instead of using additional volt- and/or ammeters in the circuit.

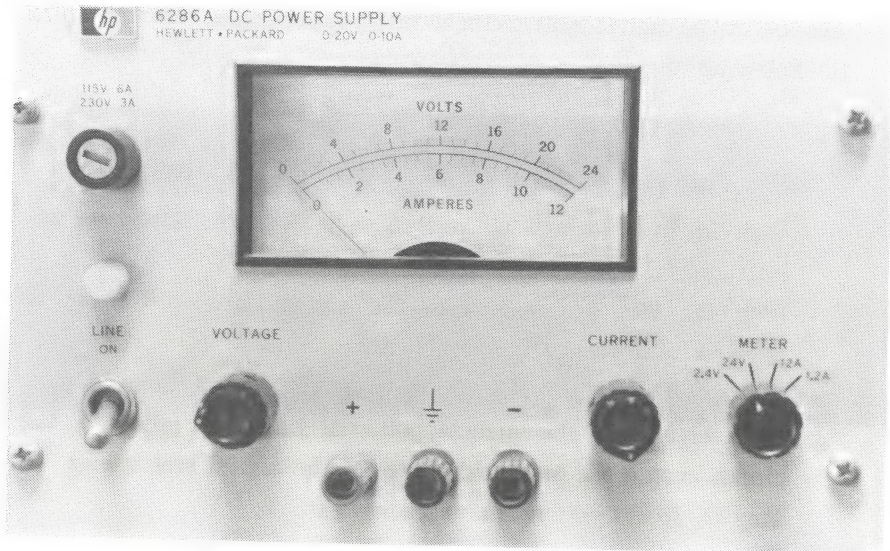


Figure 5.10 Power supply.

5.6 Cathode ray oscilloscope (CRO)

Most meters used for measuring voltages and currents involve mechanical parts. The inertia of these moving parts is much too great to permit them to follow a very rapid variation in the applied torque. These instruments do not measure instantaneous voltages and currents but rather average or effective values.

The cathode ray oscilloscope is an instrument which records instantaneous values of *rapidly varying voltages*. There is no mechanical moving part in the instrument.

The essential features of a cathode ray oscilloscope are shown in figure 5.11.

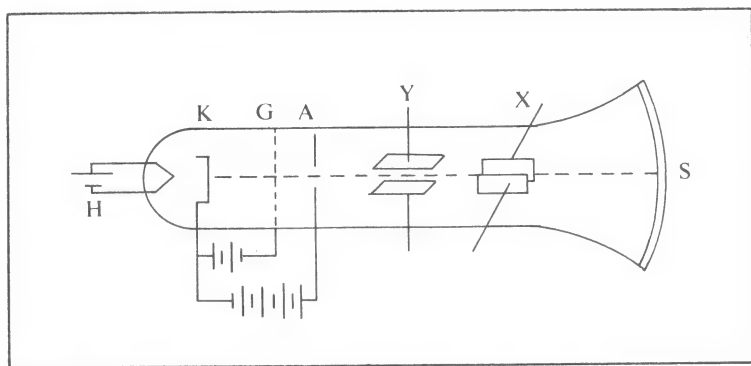


Figure 5.11 Essential features of the cathode ray oscilloscope.

The *heater* H heats the *cathode* K which emits thermionic electrons (see §10.2). These electrons are accelerated towards the *anode* A (see §10.3) which has a cylindrical form and which is maintained at a positive potential of several hundred or several thousand volts with respect to the cathode. A narrow beam of electrons is guided through the anode and eventually strikes the *screen* S. The screen consists of a thin layer of zinc sulfide coated on the evacuated glass envelope of the tube. When the screen is struck by the electrons it emits light (it fluoresces). The intensity of the light spot, i.e. the intensity of the beam of electrons is controlled by means of a *grid* G placed between the cathode and the anode. Between the anode and the screen the beam passes between two sets of *plates*, X and Y. Electric fields across these plates control the horizontal and vertical deflections of the beam. A field across the X plates produces a horizontal deflection while a

field across the vertical plates Y produces a vertical deflection. If no voltages are applied across the deflection plates, a bright spot is observed at the centre of the screen. *Never let an excessively bright spot appear on the screen. It may burn the screen and decrease the life of the tube.* If a voltage is applied across the vertical plates Y that makes the upper plate positive, the spot on the screen moves up. If we make the upper plate negative, the spot moves down. If an alternating voltage is applied, the spot will move up and down. If the frequency is higher than about 10 Hz, the retentiveness of the screen and the observer's eye cause the moving spot to appear as a continuous line.

If the wave shape of an applied voltage is to be studied, the spot must be moved horizontally too, so that the pattern may spread out on the screen. This is done by connecting the horizontal deflecting plates X to a source of voltage that rises gradually at a constant rate to a maximum and then drops suddenly to zero. This saw-tooth shaped voltage causes the beam to move horizontally across the screen at a uniform speed and to snap back to its starting point. This voltage is supplied by the *sweep circuit*. The only condition that has to be fulfilled is that the period of the saw-tooth voltage must be equal to the period of the applied voltage to be studied. To make sure that the sweep frequency is exactly the same as the frequency of the voltage being studied, it is "locked in step" by means of the synchronization. Properly adjusting the synchronization will keep the wave form stationary on the screen. An ordinary laboratory oscilloscope is shown in figure 5.12.

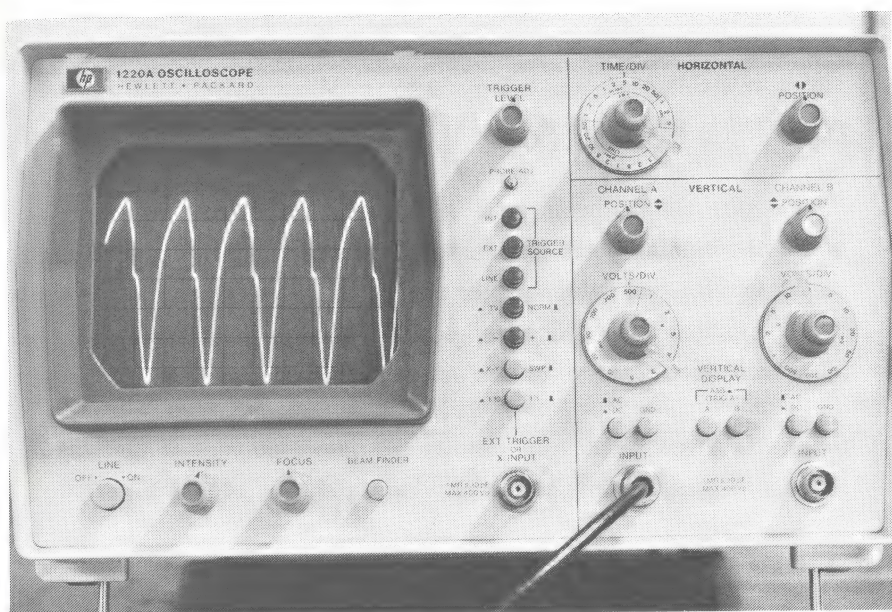


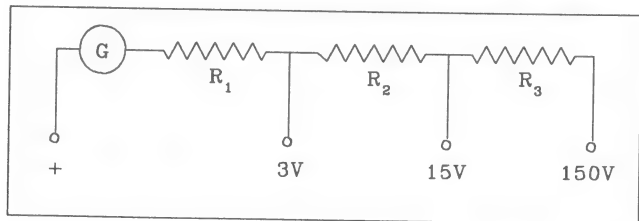
Figure 5.12 Oscilloscope.

5.7 Questions

5.1 The resistance of a galvanometer coil is $25\ \Omega$ and the current required for full scale deflection is $0.02\ \text{A}$.

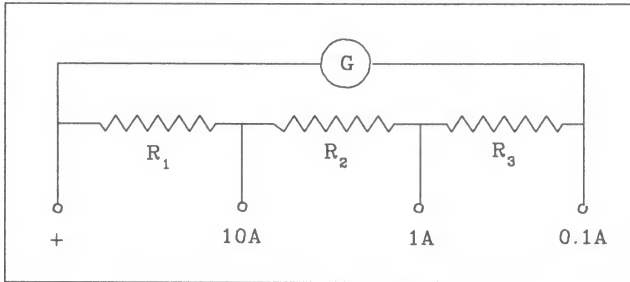
- Show in a diagram how to convert the galvanometer to an ammeter reading $5\ \text{A}$ full scale and calculate the shunt resistance.
- Show how to convert the galvanometer to a voltmeter reading $150\ \text{V}$ full scale and calculate the series resistance.

- 5.2 The resistance of the coil of a pivoted coil galvanometer is $10\ \Omega$ and a current of 0.02 A causes it to deflect full scale. It is desired to convert this galvanometer to an ammeter reading 10 A full scale. The only shunt available has a resistance of $0.03\ \Omega$. What resistance R must be connected in series with the coil?
- 5.3 The Figure shows the internal wiring of a three-scale voltmeter of which the binding posts are marked 3, 15 and 150 V. The resistance of the moving-coil galvanometer used is $15\ \Omega$. A current of 1 mA in the coil causes it to deflect full scale. Find the resistances and the total resistance of the voltmeter on each of its ranges.



- 5.4 The resistance of the moving-coil galvanometer in the ammeter shown in the figure is $25\ \Omega$ and it deflects full scale with a current of 0.01 A . Find the magnitude of the resistances required to make a multirange ammeter deflecting full scale with currents of 10 , 1 and 0.1 A respectively.

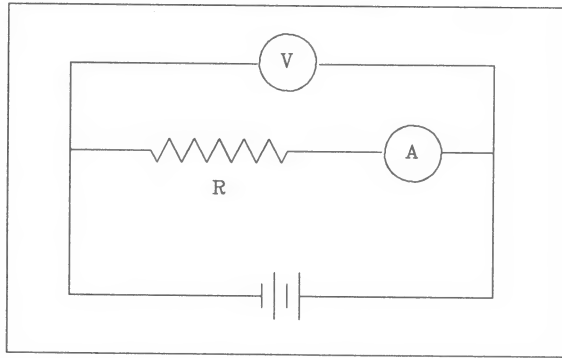
Assume that only one resistor is connected to the circuit at a time.



- 5.5 Show that if the potential difference between the anode and the cathode is kept constant, the velocity of an electron at the anode is independent of the distance between the anode and the cathode.
- 5.6 Explain how the intensity of the light spot on the screen can be controlled by the grid G in figure 5.11.
- 5.7 Explain in detail how a cathode ray oscilloscope can be used as a voltmeter.
- 5.8 Consider the circuit shown in the figure. Show that if the shunt resistance R_s of the ammeter is negligible compared to the galvanometer resistance, then,

$$R = V/I - R_s$$

where V/I is the ratio of the voltmeter reading to the ammeter reading.



5.9 Show that the deflection of the electron beam in a CRO is given by

$$d = eEL(L+2\ell)/(2mv_0^2)$$

where E is the magnitude of the electric field between a pair of plates; v_0 is the velocity of the electron as it enters the electric field; L is the length of the plates; ℓ is the distance between the plates and the screen and m and e are the mass and charge of the electron respectively.

5.10 Derive equation (5.1).

Experiments in mechanics

Chapter 6

Linear air track (exp. 1)

A body in motion or at rest must be brought into that state of motion or rest by the action of another body which in turn is brought into its state of motion or rest by a third body, and so on ad infinitum.

Benedictus de Spinoza (1632–1677)

6.1	Introduction	130
6.2	The linear air track	130
6.3	Gliders	131
6.4	Timing device	131
6.5	Levelling the air track	135
6.6	Average and instantaneous velocities	136
6.7	Precautions	136
6.8	Experiment 1(a) - analysis of the flatness of the air track	137
6.9	Experiment 1(b) - determination of the coefficient of restitution	139
6.10	Experiment 1(c) - collisions	141
6.11	Experiment 1(d) - accelerated motion	144
6.12	Questions	148

6.1 Introduction

In this series of simple experiments you will observe several basic phenomena of mechanics which you have already come across in the theoretical modules. You will gain experience in applying the principles of mechanics such as Newton's laws of motion, momentum and energy relations.

All the experiments make use of the linear air track, a simple but elegant device which permits the observation of motion with almost complete absence of friction.

6.2 The linear air track

The air track consists of a hard aluminium-alloy triangular tube P as illustrated in figure 6.1. A blower forces air through small holes on each side of the vertex of the track. These air jets support the glider(s) Q and thus almost frictionless motion is achieved. The track is also equipped with levelling screws, a centimeter scale R, spring bumpers S, photocell gates T, timers, etc.

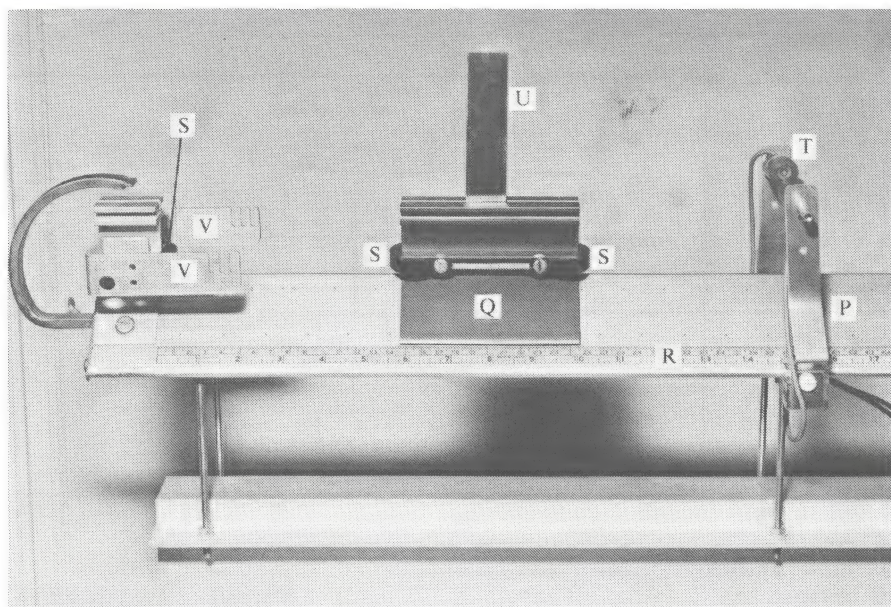


Figure 6.1 Linear air track.

6.3 Gliders

Metal gliders of different lengths and masses are available. Each glider has a spring bumper at both ends and a slot in which a piece of metal plate U, approximately $10\text{ cm} \times 2.5\text{ cm}$, can be inserted. This plate is called a flag and can be oriented either horizontally or vertically.

6.4 Timing device

A photocell gate is formed by shining a narrow beam of light on a photoelectric detector. Three photocells P_A , P_B and P_C (see figure 6.2)

and three electronic timers T_A , T_B and T_C are connected to a timer sequencer.

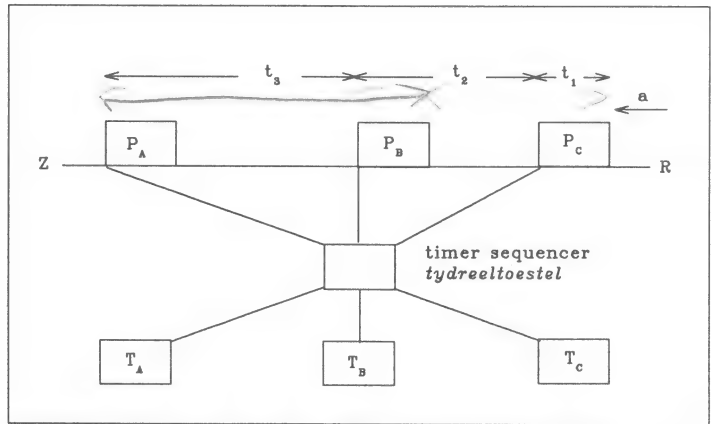


Figure 6.2 Schematic diagram of the photocells P_A , P_B and P_C and the timer sequencer.

When the flag enters the region between a light source and a photocell (a photocell gate), the illumination on the photocell changes from light to dark and one or more of the timers can be started or stopped. As soon as the flag leaves the region between a light source and a photocell, one or more of the timers can be started or stopped. There are six timer switches, labelled D, E, F, G, H and I on the front panel of the timer sequencer (see figure 6.3). Each of these switches can be set to the numbers 0, 1, 2, 3, 4 or 5.

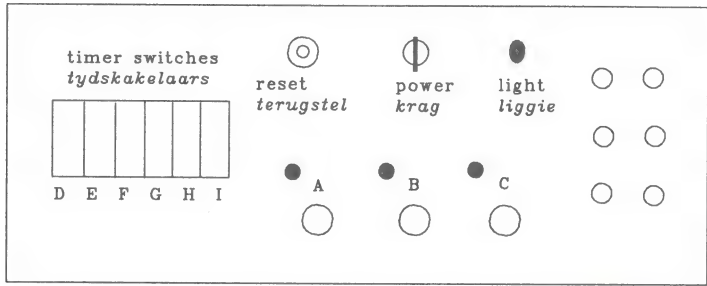


Figure 6.3 Front panel of the timer sequencer with the counter switches D to I.

As illustrated in figure 6.4, timer T_A is started by switch D and stopped by switch E. Similarly timer T_B is started by switch F and stopped by switch G, etc. If counter switch H is set to 0, timer T_C will be activated by photocell P_A when the illumination on photocell P_A changes from light to dark. Similarly, if counter switch G is set to 2, timer T_B will be stopped by photocell P_B when the illumination falling on photocell P_B changes from light to dark.

TIMER	STARTED BY	STOPPED BY
T_A	D	E
T_B	F	G
T_C	H	I

TIMER SWITCH SETTING	ACTIVATED BY PHOTOCELL	WHEN ILLUMINATION FALLING ON THE PROTOCELL CHANGES	
		FROM	TO
0	P_A	LIGHT	DARK
1	P_A	DARK	LIGHT
2	P_B	LIGHT	DARK
3	P_B	DARK	LIGHT
4	P_C	LIGHT	DARK
5	P_C	DARK	LIGHT

Figure 6.4 Timer and switch settings.

Example 6.1

Suppose a glider starts from rest at R and accelerates toward Z as shown in figure 6-2.

The idea is to measure the following time intervals:

- t_1 : The time required for the flag to move past photocell P_C .
- t_2 : The time required for the glider to move from photocell P_C to photocell P_B .
- t_3 : The time required for the glider to move from photocell P_B to photocell P_A .

Suppose we want to read times t_1 , t_2 and t_3 on timer T_A , T_B and T_C respectively. Try to determine the switch settings yourself before looking at the correct counter switch settings shown below.

4	5	4	2	2	0
D	E	F	G	H	I

Timer T_A will be started by photocell P_C when the illumination on P_C changes from light to dark, i.e. when the front of the flag reaches photocell P_C . At the same time timer T_B will be started. Timer T_A will stop as soon as the rear of the flag moves past photocell P_C . The time registered by timer T_A is the required time t_1 . When the flag reaches photocell P_B , timer T_B will be stopped and timer T_C started. The time registered on timer T_B is the time t_2 . As soon as the flag reaches photocell P_A , timer T_B will be stopped. It registers time t_3 .

6.5 Levelling the air track

It is easy to level a straight air track. Merely place a glider on the track anywhere and turn on the air supply. If the glider accelerates away from the levelling screw, use the screw to lower that end of the track. Keep adjusting the levelling screw until a stationary glider remains essentially stationary. At the other end of the track are two additional setting screws. *Don't adjust these screws.*

6.6 Average and instantaneous velocities

The *instantaneous velocity* of a glider at a point on the air track is defined as

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t} \quad (6.1)$$

if Δt is very short. If an *instantaneous velocity* is required, the narrow flag should therefore be used (vertical orientation). Since the width of the flag is then equal to Δx , Δt will be shorter than the corresponding Δt for the wider flag. There is however, a certain systematic error involved in the measurement of time intervals and since this error should not exceed a few percent of Δt , the minimum width of the flag is limited. If an *average velocity* is required a longer Δt is used, i.e. the wider flag is used (horizontal orientation). The percentage error in Δt is smaller and the average velocity more accurate.

6.7 Precautions

1. Be careful not to bump, jar or drop the gliders. Dropping a glider from a height of a few centimeters will probably ruin it.
2. Do not slide gliders on the track while the air supply is turned off.
3. If additional masses are attached to a glider, be sure to add them symmetrically. If the glider is lop-sided, it will rub on the track.
4. Do not touch the electronic timers.
5. Handle the air track carefully. Although it appears quite rigid, its use makes it extremely sensitive to small changes in alignment.

6.8 Experiment 1(a) - Analysis of the flatness of the air track

The very low friction of a glider on an air track means that in most experiments the glider can be considered part of a *conservative system*, i.e. energy considerations can be used reliably to characterise the system. Suppose the track is horizontal and almost flat, but not quite, as shown exaggerated in figure 6.5.

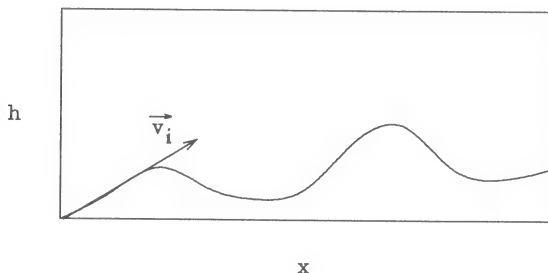


Figure 6.5 Exaggerated shape of the track.

The glider leaves the one end with an initial velocity $\vec{v}_i$. The energy equation of the glider is

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv_i^2 \quad (6.2)$$

or

$$h = (v_i^2 - v^2)/(2g) \quad (6.3)$$

where $\vec{v}$ is the velocity at a point with height h relative to that of the launching position.

Since a small change in glider height h has a one to one correspondence with an equivalent change in the value of v^2 , the flatness of the air track can be analysed by measuring the velocity $\vec{v}$ of the glider as a function of distance (x) from the starting point.

Each end of the air track is equipped with a U-shaped metal bracket V for launching gliders. Figure 6.6 shows how a rubber band W is placed between a pair of slots on the bracket arms. Stretch the rubber band until the two spring bumpers (one on the glider and one at the end of the track) are lightly in contact with the rubber band in between. If the glider is released, it is projected along the track with an initial velocity $\vec{v}_1$. To obtain reproducible velocities, the rubber band must not be coiled up and the contact position between the bumper and the rubber band must always be the same. Select a rubber band that gives an initial velocity of less than 10 cm s^{-1} .

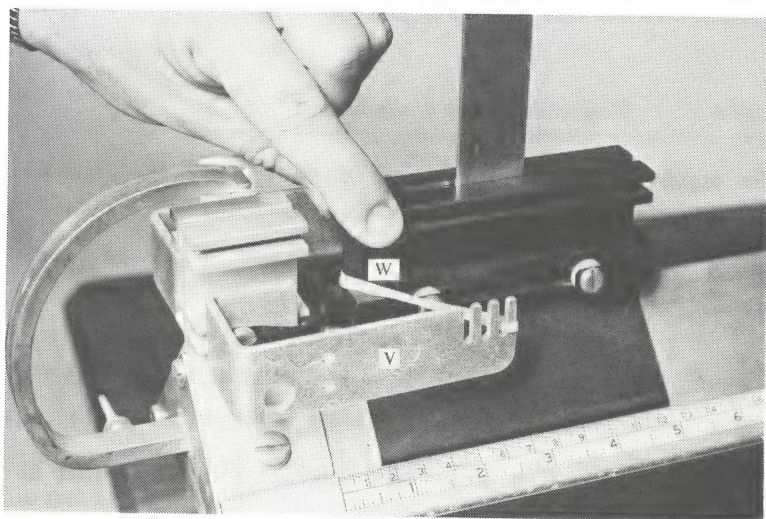


Figure 6.6 End of the air track.

By using this launching mechanism, two photocells and two timers, the initial and final velocities of the glider can be determined at a number of positions along the track. The initial velocity is determined by using a photocell positioned close to the initial position of the glider (figure 6.6). Why should this photocell not be too close to the glider?

The experiment includes the following:

1. The simplest timer switch setting is 012345. Check it to see if you agree.
2. Determine the ~~initial and final~~ ^y velocities of the glider five times for a specific value of x .
3. Repeat the measurements for 50 cm intervals of x .
4. Plot a graph of v^2 as a function of x .
5. Calculate the standard errors in the values of v^2 and indicate the errors on the graph.
6. Use the conservation of energy to calculate h as a function of x . Plot a graph. Indicate the standard error on the h values.

6.9 Experiment 1(b) – Determination of the coefficient of resitution

A useful number that can be used to characterise a rebound or collision is the coefficient of restitution e . This number is defined as the ratio of the magnitude of the relative velocity after the collision to the magnitude of the relative velocity before the collision. If the relative velocity has the same magnitude before and after the collision, the collision is *perfectly elastic* and $e = 1$. If the relative velocity has a smaller magnitude than before, the collision is *semi-elastic* and $0 < e < 1$. If the relative velocity is zero after

the collision, i.e. if the two bodies stick together, it is *completely inelastic* and $e = 0$. In this case the two bodies move as a unit after the collision.

An air track is ideal for the study of coefficients of restitution, since friction is negligible. Suppose a glider starts from rest a distance x_1 from the lower end of an air track tilted at an angle θ as shown in figure 6.7.

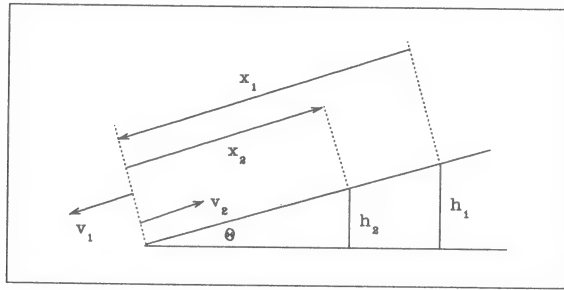


Figure 6.7 Tilted air track

According to the conservation of energy

$$\frac{1}{2}mv_1^2 = mgh_1$$

where $\vec{v}_1$ is the velocity of the glider at the lower end of the track. If the velocity immediately after the collision is $\vec{v}_2$ in the opposite direction, energy conservation yields

$$\frac{1}{2}mv_2^2 = mgh_2 .$$

Since

$$\sin \theta = h_1/x_1 = h_2/x_2 ,$$

$$v_1 = (2gx_1 \sin \theta)^{\frac{1}{2}} \quad (6.4)$$

and

$$v_2 = (2gx_2 \sin \theta)^{\frac{1}{2}}. \quad (6.5)$$

The coefficient of restitution is therefore given by

$$\begin{aligned} e &= v_2/v_1 \\ &= (x_2/x_1)^{\frac{1}{2}} \end{aligned} \quad (6.6)$$

The experiment includes the following:

1. Tilt the track by placing a suitable object under the levelling screw. Note that the value of θ is not required.
2. Determine values of x_2 five times for each of seven different values of x_1 .
3. Draw a straight line graph by using a linear regression analysis and determine e from the slope of the graph.
4. Calculate s_e .

3.10 Experiment 1(c) – Collisions

The law of conservation of linear momentum states that the total linear momentum of an *isolated system* is constant in magnitude and direction. The term isolated implies that *no resultant external force is exerted on the system*. Consider a collision between two gliders a and b. Before the

collision the velocities are $\vec{v}_{a1}$ and $\vec{v}_{b1}$ respectively and after the collision $\vec{v}_{a2}$ and $\vec{v}_{b2}$ respectively. Since no resultant external force is exerted on the gliders, linear momentum is conserved. Therefore

$$m_a \vec{v}_{a1} + m_b \vec{v}_{b1} = m_a \vec{v}_{a2} + m_b \vec{v}_{b2} . \quad (6.7)$$

This equation is valid whether the collision is elastic or inelastic. If the collision is perfectly elastic the total kinetic energy of the system is also constant:

$$\frac{1}{2}m_a v_{a1}^2 + \frac{1}{2}m_b v_{b1}^2 = \frac{1}{2}m_a v_{a2}^2 + \frac{1}{2}m_b v_{b2}^2 . \quad (6.8)$$

If the collision is not perfectly elastic, the total kinetic energy after the collision is less than the total kinetic energy before the collision.

The experimental setup is shown in figure 6.8. Three photocells P_A , P_B and P_C are arranged on the air track with the distance between adjacent photocells not more than 30 cm. If you have done Experiment 1(a), you should consult the graph of h versus x to select the best section of the track for this experiment.

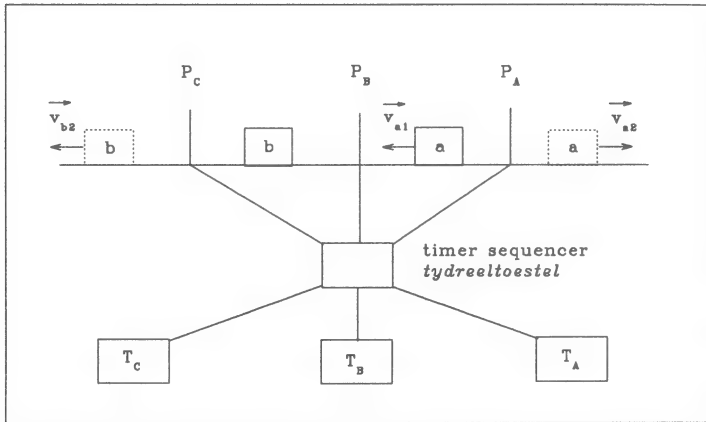


Figure 6.8 Arrangement of photocells for experiment 1(c).

Glider a is given an initial velocity $\vec{v}_{a1}$ between photocells P_A and P_B . Timer T_A is started as soon as the illumination on photocell P_B changes from light to dark and is stopped when the illumination changes from dark to light. From the time registered on timer T_A and the length of the flag, $\vec{v}_{a1}$ can be calculated.

The collision takes place between photocells P_B and P_C . Glider b, which was stationary before the collision ($\vec{v}_{b1} = 0$), moves towards photocell P_C and from the time registered on timer T_C , $\vec{v}_{b2}$ can be calculated. If the collision is perfectly inelastic, the two gliders stick together after the collision and $\vec{v}_{b2} = \vec{v}_{a2}$. If the collision is elastic, glider a moves towards photocell P_A after the collision and the final velocity $\vec{v}_{a2}$ is calculated from the time registered on timer T_B . The masses are indicated on the gliders.

The experiment includes the following:

1. The simplest timer switch setting is 012345. Check it to see if you agree.
2. Investigate three elastic collisions between two gliders with different masses.
3. Compare the total momentum (a vector) and kinetic energy (a scalar) before a collision with the corresponding values after the collision. *for each collision independently*
4. Repeat the experiment with two identical gliders.
5. Repeat (2) and (3) for inelastic collisions. An inelastic collision is obtained by putting small pieces of "Pres-Stik" on the bumpers of the gliders.

6.11 Experiment 1(d) – Accelerated motion

If the air track makes an angle θ with the horizontal, a resultant force is exerted on the glider as shown in figure 6.9.

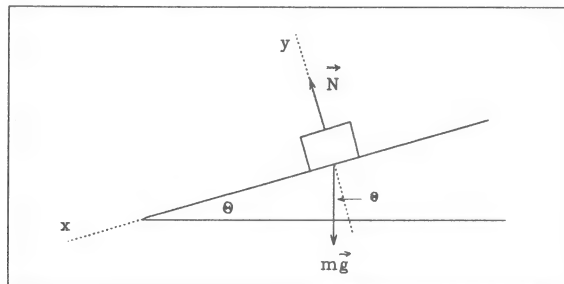


Figure 6.9 Forces on the glider

Two forces are exerted on the glider, viz. the force $m\vec{g}$ exerted on the glider by the earth, and the force $\vec{N}$ exerted by the air on the glider. The sum of the forces perpendicular to the track is

$$\Sigma \vec{F}_y = \vec{N} - m\vec{g}\cos\theta$$

and since the glider is not accelerated in this direction (the track is assumed to be perfectly straight), it follows from Newton's second law that

$$\Sigma \vec{F}_y = 0.$$

Therefore

$$\vec{N} = m\vec{g}\cos\theta.$$

The sum of the forces along the surface of the track is

$$\Sigma \vec{F}_x = m\vec{g}\sin\theta$$

and according to Newton's second law the glider will have an acceleration a_x where

$$\begin{aligned} m\vec{a}_x &= \Sigma \vec{F}_x \\ &= m\vec{g}\sin\theta. \end{aligned}$$

Therefore

$$\vec{a}_x = \vec{g}\sin\theta.$$

The motion of the glider is described by

$$\begin{aligned} v_x &= v_{ox} + a_x t \\ &= v_{ox} + gt\sin\theta, \end{aligned} \tag{6.9}$$

$$x = v_{ox}t + \frac{1}{2}a_x t^2$$

$$= v_{ox} t + \frac{1}{2} g t^2 \sin \theta \quad (6.10)$$

and

$$\begin{aligned} v_x^2 &= v_{ox}^2 + 2a_x x \\ &= v_{ox}^2 + 2gx \sin \theta \end{aligned} \quad (6.11)$$

where v_{ox} is the magnitude of the velocity of the glider when $t = 0$.

The experimental arrangement is shown in figure 6.10.

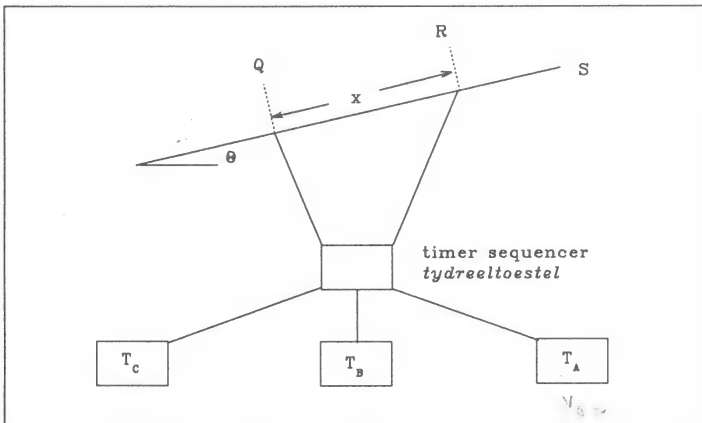


Figure 6.10 Experimental arrangement for experiment 1(d).

The glider starts from rest at position S with the narrow flag. When the illumination on photocell P_A changes from light to dark, timers T_A and T_B are activated and when the illumination changes from dark to light, timer T_A stops. The instantaneous velocity of the glider at R can therefore be calculated. A change of illumination on photocell P_B from light to dark

stops T_B and activates T_C . When the illumination changes from dark to light timer T_C stops. From the time recorded on timer T_C the instantaneous velocity at Q can be calculated. Timer T_B gives the time of travel from R to Q. The distance between R and Q can be read off the centimeter scale.

The experiment includes the following:

1. A possible timer switch setting is 010223. Check it to see if you agree.
2. Level the track and put the aluminium block under the setting screw.
3. Calculate θ from the thickness of the block and the distance between the setting screws near the end of the glider.
4. Record at least seven sets of readings of v_{ox} , v_x , x and t by changing the position of photocell P_B and keeping S and R fixed.
5. Draw a graph of v_x as a function of t and obtain v_{ox} and g from it.
6. Draw a graph of v_x^2 versus x and obtain v_{ox} and g from it.
7. Draw a graph of x as a function of t and obtain an estimate of v_{ox} from it.
8. Compare the values of v_{ox} obtained from (5), (6), (7) and the direct measurements.
9. Compare the values of g obtained from (5) and (6) with the generally accepted value of 9.8 m s^{-2} .

Please note that for all straight line graphs a linear regression analysis must be done and the errors on all required values must be calculated.



6.12 Questions

- 6.1 With what force will the feet of a passenger weighing 500 N press downward on the elevator floor when the elevator has the following accelerations:
- a) 2 m s^{-2} upward?
 - b) 2 m s^{-2} downward?
- 6.2 A body of mass 25 kg rests on a frictionless horizontal plane and is acted on by a horizontal force of 30 N.
- a) What acceleration is produced?
 - b) How far will the body travel in 10 s?
 - c) What will be its velocity at the end of 10 s?
- 6.3 Discuss the effect of friction on the results of Experiment 1(a).
- 6.4 Describe the transfer of energy when the glider is launched by using a rubber band (Experiment 1(a)).
- 6.5 Discuss the effect of friction on the results of Experiment 1(b).
- 6.6 A man and a boy initially at rest on frictionless horizontal ice, push each other apart. Which skater is further from the starting point after a short time? Explain.

- 6.7 A horse is pulling a heavy cart and the cart and the horse are both being accelerated. Is the force of the cart on the horse equal and opposite to that of the horse on the cart? If your answer is "yes", explain why these equal and opposite forces give rise to the "net force" that is necessary to cause acceleration. If your answer is "no", explain whether Newton's third law is true while the cart is accelerating.
- 6.8 Why does a glider move at constant velocity on a horizontal air track?
- 6.9 Consider two identical gliders on a horizontal air track. The one moves towards the other one which is stationary and then they collide elastically. By using conservation laws, determine what happens after the collision. What would be the effect of a tilted track on the results? Explain.
- 6.10 A mass m_1 is given an initial velocity v_0 while mass m_2 ($\neq m_1$) is initially at rest. After a semi-elastic collision the velocities are $\vec{v}_1$ and $-\vec{v}_2$ respectively. Show that the ratio of final to initial kinetic energy is given by

$$R = (m_1 + e^2 m_2) / (m_1 + m_2).$$

Use this expression to show that kinetic energy is conserved only when $e = 1$.

- 6.11 A man stands on an open truck which can run on frictionless horizontal rails. The man and the truck are at rest. The man now walks to the one end of the truck and stops. Describe the motion of the truck.

- 6.12 A 0.006 kg bullet is fired horizontally into a 4 kg wooden block resting on a horizontal surface. The coefficient of sliding friction between the block and surface is 0.25. The bullet remains embedded in the block, which is observed to slide 0.35 m along the surface. What was the velocity of the bullet?
- 6.13 On a frictionless table, a 4 kg block moving at 5 m s^{-1} to the right, collides with a 9 kg block moving at 1.6 m s^{-1} to the left.
- If the two blocks stick together, what is the final velocity?
 - If the two blocks have a completely elastic head-on collision, what are the final velocities?
 - How much mechanical energy is converted into heat in the collision of part (a)?
- 6.14 An elevator and its load weigh a total of 12.2 kN. Find the tension in the supporting cable when the elevator, originally moving downward at 10 m s^{-1} , is brought to rest with constant acceleration in a distance of 15 m?
- 6.15 A body A with mass m_1 moves at a velocity $\vec{v}_{A1}$ and collides elastically with body B, mass m_2 , which is stationary. Show that the magnitudes of the velocities of A and B after the collision are given by:

$$v_{A2} = (m_2 - m_1)v_{A1} / (m_1 + m_2)$$

and

$$v_{B2} = 2m_1 v_{A1} / (m_1 + m_2).$$

6.16 The following results were obtained for experiment 1(a):

x (cm)	t_A (s)					t_B (s)				
	1	2	3	4	5	1	2	3	4	5
0	0.094	0.100	0.102	0.106	0.098	0.135	0.140	0.136	0.144	0.145
50	0.095	0.100	0.097	0.093	0.090	0.282	0.290	0.280	0.292	0.286
100	0.099	0.108	0.114	0.111	0.110	0.144	0.136	0.141	0.140	0.139
150	0.100	1.103	0.110	0.108	0.105	0.083	0.090	0.080	0.078	0.083
200	0.100	0.101	0.102	0.098	0.104	0.071	0.070	0.076	0.069	0.067
250	0.090	0.095	0.091	0.087	0.092	0.095	0.091	0.091	0.091	0.087
300	0.091	0.090	0.099	0.095	0.100	0.122	0.125	0.126	0.128	0.124

The width of the flag is 2 cm.

Draw all graphs required for this experiment and perform all the calculations as specified in § 6.8.

6.17 The following results were obtained for experiment 1(b):

x_1 (cm)	x_2 (cm)				
10	6.4	6.3	6.5	6.6	6.2
50	32.0	33.3	30.7	32.0	31.0
70	44.0	44.8	45.6	44.0	43.1
100	65.2	66.3	62.0	63.8	64.2
150	97.0	98.1	94.0	97.3	95.0
200	130.0	133.1	128.5	129.9	130.4
250	158.0	163.3	163.8	160.3	159.9

Draw the required graph and calculate e and s_e .

6.18 The following results were obtained for experiment 1(c):

a) Elastic collision with $m_1 \neq m_2$

t_A (s)	t_C (s)	t_B (s)
1.010	2.300	1.800
0.511	1.220	0.880
0.336	0.790	0.570

$$m_1 = 0.2 \text{ kg}$$

$$m_2 = 0.5 \text{ kg (stationary before collision)}$$

$$\text{width of flag} = 0.10 \text{ m}$$

b) Elastic collision with $m_1 = m_2$

t_A (s)	t_C (s)	t_B (s)
1.010	1.030	20.010
0.511	0.512	25.600
0.336	0.341	13.660

$$m_1 = 0.2 \text{ kg}$$

$$m_2 = 0.2 \text{ kg}$$

$$\text{width of flag} = 0.10 \text{ m}$$

c) Inelastic collision with $m_1 \neq m_2$

t_A (s)	t_C (s)	t_B (s)
1.010	3.520	-
0.511	1.720	-
0.336	1.180	-

$$m_1 = 0.2 \text{ kg}$$

$$m_2 = 0.5 \text{ kg}$$

$$\text{width of flag} = 0.10 \text{ m}$$

d) Inelastic collision with $m_1 = m_2$

t_A (s)	t_C (s)	t_B (s)
1.010	2.040	-
0.511	1.080	-
0.336	0.680	-

$$m_1 = 0.2 \text{ kg}$$

$$m_2 = 0.2 \text{ kg}$$

$$\text{width of flag} = 0.10 \text{ m}$$

Do all the calculations required for this experiment.

6.19

The following results were obtained for experiment 1(d):

Thickness of block = 2 cm

Distance between setting screws = 2.5 m

Width of flag = 2.0 cm

x (cm)	t_A (s)	t_B (s)	t_C (s)
10	0.412	15.400	0.0160
50	0.408	36.010	0.0077
100	0.404	50.020	0.0059
150	0.414	61.010	0.0049
190	0.400	69.341	0.0030
210	0.422	74.009	0.0033
250	0.410	78.099	0.0038

$$\frac{\Delta v}{v} = \frac{v_2 - v_1}{v_1} = 0$$

Draw all the required graphs and do all the calculations specified in § 6.11.

Chapter 7

Uniform Circular Motion (exp. 2)

I have yet to see any problem, however complicated, which when you look at it the right way does not become still more complicated

P Anderson

7.1	Introduction	155
7.2	Experiment 2(a) - centripetal force (I)	157
7.3	Experiment 2(b) - centripetal force (II)	162
7.4	Questions	166

7.1 Introduction

While motion in only one dimension, i.e. rectilinear motion, was investigated in chapter 6, curved motion will be studied in this chapter. A common example of curved motion which is very important and yet not difficult to analyse is uniform circular motion; i.e. motion in a circle at constant speed.

The *magnitude* of the instantaneous acceleration of a particle executing uniform circular motion is equal to the square of the speed divided by the radius.

$$a = v^2/R. \quad (7.1)$$

The *direction* of the acceleration is inward along the radius towards the centre of the circle. Because of this, it is called a central, a centripetal or a radial acceleration. According to Newton's second law the magnitude of the force exerted on the particle is

$$\begin{aligned} F &= ma \\ &= mv^2/R. \end{aligned} \quad (7.2)$$

The direction of this force is the same as the direction of the acceleration, i.e. towards the centre of the circle. It is unfortunate that it has become common practice to characterise the force by the adjective "centripetal", since this seems to imply that there is some difference in nature between centripetal forces and other forces. This is not the case. Centripetal forces, like other forces, are exerted by sticks and strings or arise from the action of gravitational or other causes. The term "centripetal" refers to the effect of

the force, that is to the fact that it results in a change in the direction of the velocity of the body on which it acts rather than a change in the magnitude of this velocity.

A number of examples of uniform circular motion have been listed in Table 7.1. Note that there is always a resultant force directed towards the centre of the circle, exerted on the body executing the motion.

Table 7.1 Examples of uniform circular motion

System	No. of forces	Nature of force	Direction	Magnitude
Moon revolving about the earth	1	Gravitational	Towards earth	Gm_em_m/r^2
Electron in hydrogen atom	1	Electrostatic	Towards proton	kq_eq_p/r^2
Charged particle moving in magnetic field	1	Magnetic	Towards centre	$qvB\sin\theta$

Centrifugal force
is not a force

Frequently students wish to add an outward "centrifugal" force "to keep the body out there". To look for such a force is unnecessary because the body does not stay out there. *Its velocity changes all the time* and according to Newton's first law it is not in equilibrium. If an outward force were acting on it, equal and opposite to the "centripetal" force, there would be no resultant inward force to change its velocity and the body would move along a straight line.

7.2 Experiment 2(a) - centripetal force (I)

According to the theory of uniform circular motion, the magnitude of the resultant force exerted on a body executing circular motion is mv^2/R and this force is always directed towards the centre of the circle. The purpose of this experiment is to investigate this result by exerting a known force on the body and to measure the speed of the body and the radius of the circle.

The apparatus, shown in figure 7.1, consists of a hollow transparent cylinder A in which a spiral spring B and a steel sphere C are placed. The inner diameter of the cylinder is slightly greater than the diameters of the coil spring and the sphere. The length of the spring is indicated by a pointer D on a centimeter scale on the outside of the cylinder. If the spring is compressed and then allowed to extend again, the pointer remains in the position of maximum compression. To obtain a new reading, the pointer must always be returned manually to the zero position on the cylinder. The cylinder is mounted on a horizontal aluminium bar E which can rotate at different angular velocities about a vertical axis F.

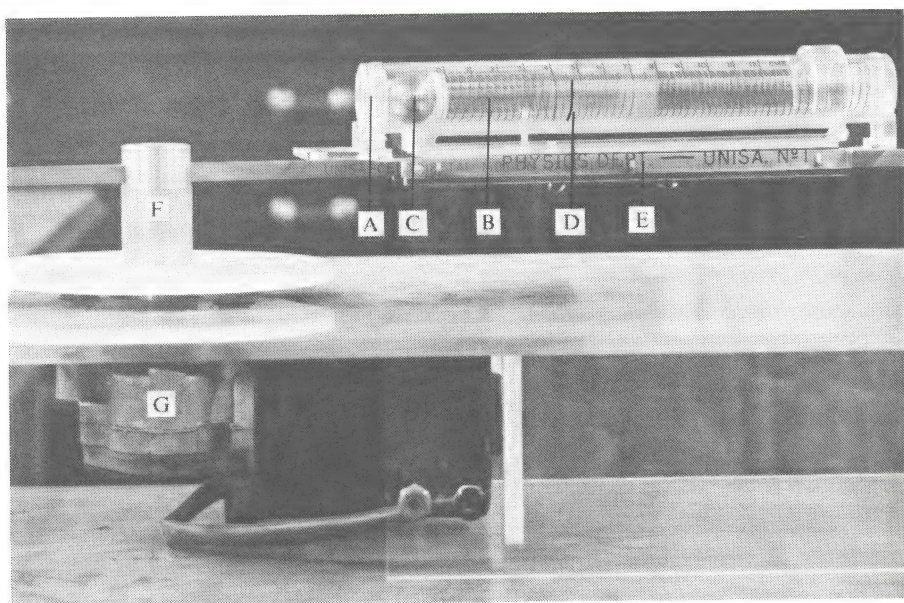


Figure 7.1 Uniform circular motion apparatus I.

The distance of the zero position on the cylinder from the axis of rotation can be read off a scale on the aluminium bar. The angular velocity can be changed by changing the power supplied to the electric motor G which drives the system.

All the forces acting *on the sphere* when the apparatus is *stationary* are shown in figure 7.2.

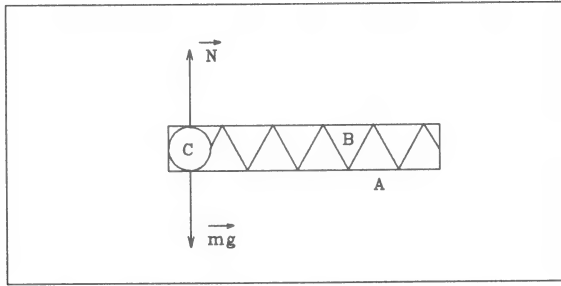


Figure 7.2 Forces on the sphere when the apparatus is stationary.

If it is assumed that the spring has its natural length, it does not exert a force on the sphere. $\vec{N}$ is the force exerted by the cylinder on the sphere and $\vec{mg}$ is the force exerted by the earth on the sphere. Please note that since the sphere is in equilibrium,

$$\begin{aligned}\Sigma \vec{F} &= \vec{N} + \vec{mg} \\ &= 0\end{aligned}$$

and therefore

$$\vec{N} = -\vec{mg}. \quad (7.3)$$

Nevertheless, $\vec{N}$ and $\vec{mg}$ do not form an action–reaction pair. Newton's third law states that if object 1 exerts a force on object 2, then object 2 exerts an equal and opposite force on object 1. It is therefore clear that the reaction to $\vec{N}$ is the downward force, magnitude N , exerted by the sphere on the cylinder,

$$\vec{N}(\text{cylinder on sphere}) = -\vec{N}(\text{sphere on cylinder}).$$

Similarly the reaction to mg is an upward force, magnitude mg , exerted by the sphere on the earth,

$$m\vec{g}(\text{earth on sphere}) = -m\vec{g}(\text{sphere on earth}).$$

In general Newton's third law can be stated mathematically as:

$$\vec{F}_{12} = -\vec{F}_{21}. \quad (7.4)$$

If the apparatus *rotates* at constant angular velocity, the sphere executes circular motion and the resultant inward force (centripetal force) $\vec{F}$ is exerted on the sphere by the spring. The force diagram for this situation is shown in figure 7.3.

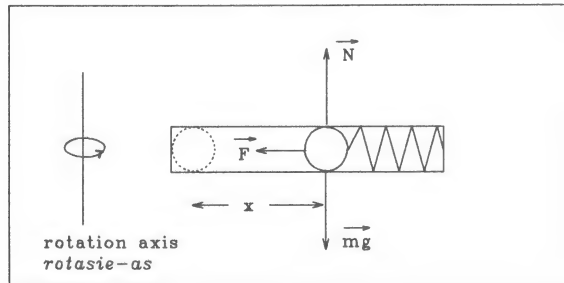


Figure 7.3 Force diagram when the apparatus rotates.

According to Hooke's law

$$\vec{F} = -k\vec{x} \quad (7.5)$$

where k is the spring constant and $\vec{x}$ is the *displacement* of the sphere along the axis of the spring, i.e. in this case $\vec{x}$ is the compression of the spring. The fact that $\vec{F}$ and $\vec{x}$ are in opposite directions accounts for the negative sign. According to Newton's third law

$$\vec{F}(\text{spring on sphere}) = -\vec{F}(\text{sphere on spring}).$$

The sphere therefore exerts a force, magnitude F , on the spring in the outward direction, i.e. away from the rotation.

The magnitude of the resultant force on the sphere towards the centre of motion is therefore kx . The theory of circular motion predicts that this force can always be represented by mv^2/R , i.e.

$$mv^2/R = kx$$

or

$$v^2/R = kx/m. \quad (7.6)$$

If we determine v , R and x experimentally and find that equation (7.6) is valid, we know that the centripetal force can indeed be represented by mv^2/R .

The experiment includes the following:

1. Reset the pointer to zero, start the motor and determine the period of the circular motion. It is suggested that the time for at least 30 revolutions be measured.

2. Record the distance a from the axis of rotation to the zero position of the x scale. This value is read off the scale on the aluminium bar. Keep this distance fixed (10 cm).
3. Record the compression of the spring (x) corresponding to the period measured in step (1).
4. Repeat steps (1), (2) and (3) for at least seven different periods varying the power supplied to the electric motor.
5. Calculate the radius, $R = a + x$, and speed of each set of readings.
6. Plot a suitable graph to verify that the centripetal force equals mv^2/R and determine the ratio k/m from the graph.
7. Use a linear regression analysis to determine the error on your result.

7.3 Experiment 2(b) – centripetal force (II)

The purpose of this experiment is to prove the existence of the centripetal force and to investigate its magnitude and direction by using the apparatus shown in figure 7.4.

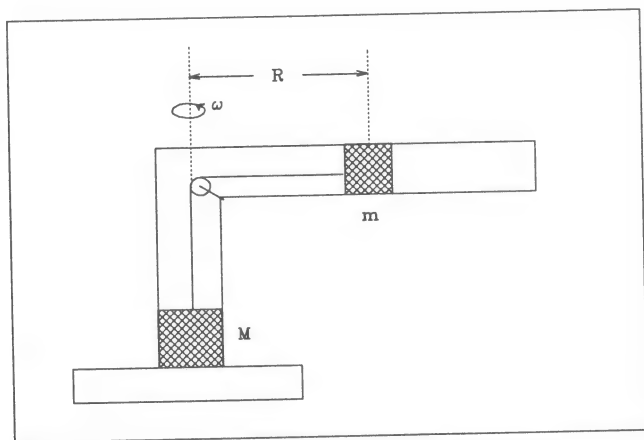


Figure 7.4

Circular motion apparatus II.

A cylindrical object, mass m , in a horizontal transparent cylinder, is connected with a thin wire to another cylindrical object, mass M , resting on the floor of a vertical transparent cylinder ($M = 2m$). An electric motor rotates the apparatus about the vertical axis at an angular velocity ω which can be varied by changing the power supplied to the motor.

If the apparatus is *stationary*, the tension in the wire is zero and the forces acting on the objects are shown in figure 7.5.

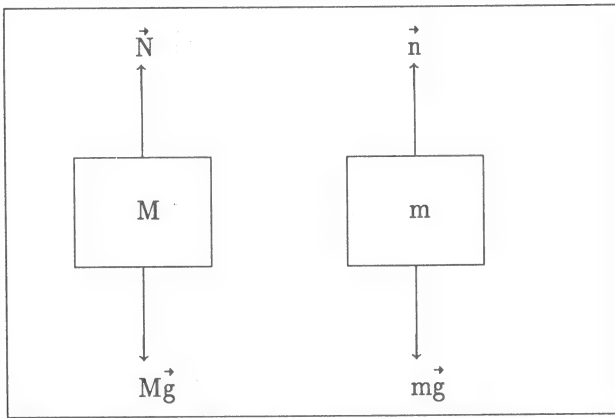


Figure 7.5 Force diagram for stationary apparatus.

$M\vec{g}$ and $m\vec{g}$ are the forces exerted by the earth on M and m respectively while $\vec{N}$ and $\vec{n}$ are the forces exerted by the cylinders on the objects. What are the reactions to these forces?

When the apparatus is *rotating*, an inward force is exerted by the wire on the small object. This force is the centripetal force required for the circular motion of m . If the speed of m is low, both objects will rest on the cylinders and the forces acting on m and M are those shown in figure 7.6.

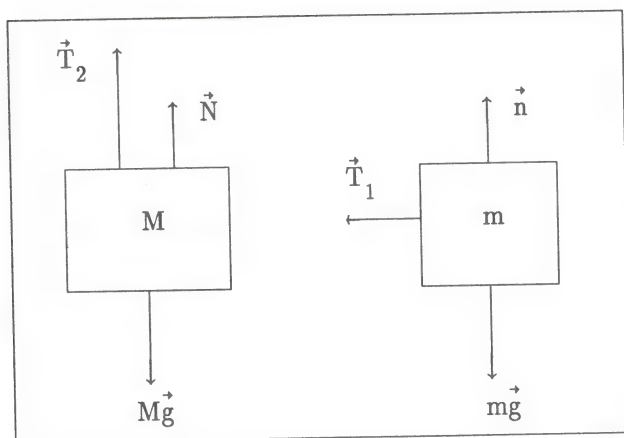


Figure 7.6 Force diagram for rotating apparatus.

$\vec{T}_1$ is the force exerted by the wire on m and if we assume that the pulley is frictionless, the wire exerts an upward force $\vec{T}_2$ on M . If the mass of the wire is much less than m or M , $\vec{T}_1 = \vec{T}_2 = \vec{T}$. $M\vec{g}$, $m\vec{g}$ and $\vec{n}$ are equivalent to the corresponding forces shown in figure 7.5. However, if Newton's first law is applied to M , it follows that

$$\vec{T} + \vec{N} = M\vec{g},$$

and therefore

$$\vec{N} = M\vec{g} - \vec{T},$$

whereas if the apparatus is stationary

$$\vec{N} = M\vec{g}.$$

Since we don't know the magnitude of $\vec{N}$ when the apparatus is rotating, we cannot say anything about $\vec{T}$. However, if the power supplied to the motor is carefully adjusted, a situation is reached where M is in equilibrium

without touching the bottom of the vertical cylinder. The forces acting on the objects in this case, are shown in figure 7.7. Once again we assume that

$$\vec{T}_1 = \vec{T}_2 = \vec{T}.$$

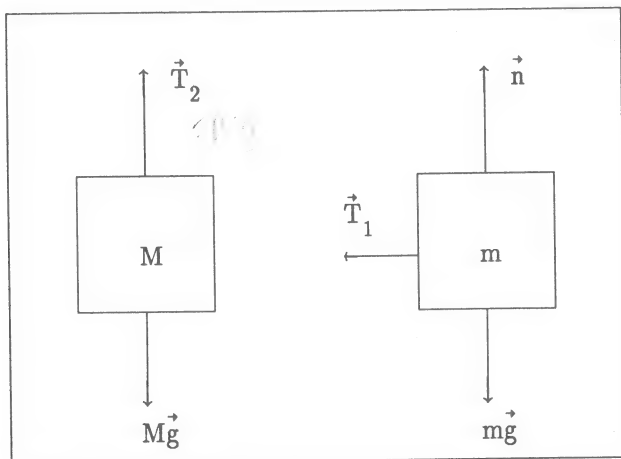


Figure 7.7 Force diagram when M is not touching the bottom.

Since M is in equilibrium,

$$\vec{T} - M\vec{g} = 0$$

$$\therefore \vec{T} = M\vec{g}.$$

The inward force exerted by the wire on m is therefore also equal to $M\vec{g}$. From the theory of circular motion, it follows that

$$mv^2/R = M\vec{g}. \quad (7.7)$$

If the radius of the circle and the speed of m are determined, M can be calculated and compared with the known value. Agreement between these

values would support the theory of circular motion which predicts that the magnitude of the centripetal force is equal to mv^2/R .

The experiment includes the following:

1. Adjust the speed of m until M is in equilibrium and not more than 0.2 cm from the bottom of the vertical cylinder.
2. Determine the period of motion.
3. Calculate the speed of m by using the radius given on the apparatus.
4. Calculate M .
5. Repeat steps (1), (2), (3) and (4) five times and calculate the average value of M and s_M . Compare this value with the one given on the apparatus.
6. Draw force diagrams similar to those in figures 7.5, 7.6 and 7.7 and indicate all the reactions to these forces.

7.4 Questions

- 7.1 What keeps an earth satellite moving in its orbit? What keeps the satellite up? Are your answers applicable to the moon?
- 7.2 Show how the period of free oscillation of the sphere and spring used in Experiment 2(a) can be obtained from the slope of the graph.
- 7.3 Is the steel sphere in equilibrium when the apparatus is rotating at constant angular velocity? Explain.
- 7.4 Is it possible for a body to be accelerated if its speed is constant? Explain.

7.5 Consider an aluminium atom near the end of the aluminium bar. Does any centripetal force act on this atom if the apparatus rotates? If so, what is this force caused by? *Draw diagram atom being forced in centre*

7.6 "The earth does not fall into the sun because a centrifugal force pushes it outwards". Comment on this statement.

7.7 Why is it hard to find an equilibrium position for mass M if it does not touch the bottom of the vertical cylinder?

7.8 The radius of a hydrogen atom (Bohr radius) is 5.29×10^{-11} m.

- Find the electrostatic force between the proton and electron.
- How fast does the electron move if it is orbiting the proton at this radius?
- How many revolutions per second does the electron complete?

7.9 How many forces act on the electron in the hydrogen atom?

7.10 A car is going around a circular track of radius 100 m with a constant speed of 20 m s^{-1} . What is the centripetal acceleration at the instant when the car is due north from the centre of the circle and travelling east? Draw a diagram showing the path, the velocity vector and the acceleration vector.

7.11 Just after the instant mentioned in question 7.10, the car acquires a tangential component of acceleration equal to 3 m s^{-2} . Draw a diagram showing the path, the tangential and centripetal components of the acceleration. Find the magnitude and direction of the acceleration vector.

- 7.12 Discuss the effects of friction on the results of the experiments. Draw diagrams similar to those shown in figures 7.2, 7.3, 7.5, 7.6 and 7.7 indicating the frictional forces as well. Give reasons for the choice of direction of the frictional forces.
- 7.13 Suppose the aluminium bar moves in a horizontal plane but the cylinder is not perfectly horizontal. Discuss the effect of this misalignment on the result.
- 7.14 Does the mass of the spring influence the results?
- 7.15 Why is it important not to *decrease* the speed of the sphere before the period is determined?
- 7.16 Is it possible to estimate the speed of the steel sphere if the power dissipated by the electric motor is known?
- 7.17 The following results were obtained for experiment 2(a):

$$a = 10 \text{ cm}$$

x (m)	Time for 30 revolutions (s)
0.01	280.4
0.05	147.2
0.10	122.6
0.15	106.8
0.20	104.6
0.25	99.7
0.30	96.9

Analyse these results as indicated in § 7.2.

7.18 The following results were obtained for experiment 2(a):

$$a = 10 \text{ cm}$$

x (m)	Time for 30 revolutions (s)
0.02	268
0.04	205
0.07	170
0.12	146
0.18	134
0.23	129
0.25	128
0.32	125

Analyse these results as indicated in § 7.2.

7.19 The following results were obtained for experiment 2(a):

$$a = 10 \text{ cm}$$

x (m)	Time for 30 revolutions (s)
0.02	154
0.04	116
0.07	98
0.12	84
0.18	77
0.23	76
0.25	74
0.32	72

Analyse these results as indicated in § 7.2.

$$v = \frac{2\pi r}{T} \quad (\text{circular motion})$$

$$F_{\text{spring}} : F_{\text{centrifugal}} \Rightarrow -kx = \frac{mv^2}{R}$$

$$\left(\frac{k}{m}\right) x = \frac{v^2}{R}$$



Chapter 8

Simple harmonic motion (exp. 3)

An intelligence knowing all forces acting in nature as well as the momentary positions and velocities of all things of which the universe consists, would be able to comprehend the motions of the largest bodies of the world and those of the lightest atoms; to him nothing would be uncertain, both present and future would be in his eyes.

Pierre de Laplace (1749–1827)

8.1	Introduction	171
8.2	Simple harmonic motion in a straight line	171
8.3	Experiment 3(a) - Simple harmonic motion of a mass hanging on an idealized spring	174
8.4	Experiment 3(b) - The simple pendulum	176
8.5	Questions	178

8.1 Introduction

When a particle moves about a fixed point O in such a way that (a) its acceleration vector always points at O and (b) the magnitude of the acceleration is proportional to the distance between the particle and O , then the particle executes *simple harmonic motion (SHM)*. Read more about this subject in your first year textbook.

8.2 Simple harmonic motion in a straight line

Consider a particle P moving in a circle of radius r with constant speed v_p about the centre O (figure 8.1).

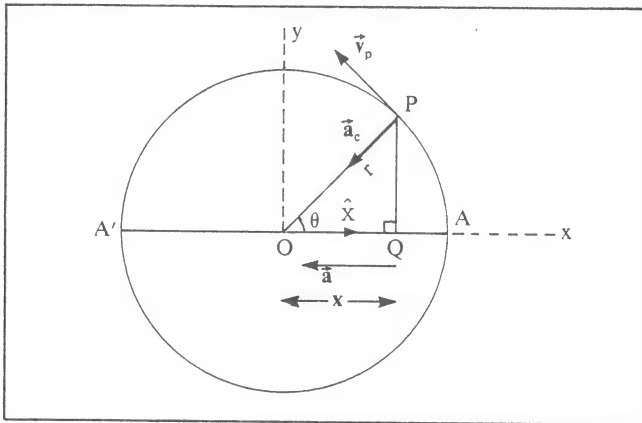


Figure 8.1 Particle P moving in a circle of radius r .

Suppose AOA' is any diameter of the circle and that another particle Q is moving along AOA' in such a manner that PQ is always perpendicular to the diameter AOA' . As P describes the circle, Q oscillates to and fro along the diameter. The magnitude of the centripetal acceleration of P is

$$a_c = v_p^2 / r . \quad (8.1)$$

In terms of the angular velocity $\omega (= v_p / r)$ of P the magnitude of the acceleration of P is

$$a_c = \omega^2 r . \quad (8.2)$$

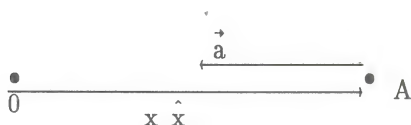
This acceleration is directed towards the centre O. The acceleration $\vec{a}$ of particle Q must equal the components of $\vec{a}_c$ in the x direction.

Therefore

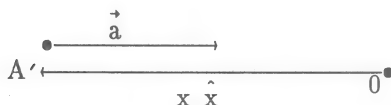
$$\begin{aligned} \vec{a} &= \omega^2 r \cos(180^\circ - \theta) \hat{x} \\ &= - \omega^2 r \cos \theta \hat{x} \\ &= - \omega^2 x \hat{x} \end{aligned}$$

where x is the distance of the particle Q from the centre O. Since the displacement of the particle from the centre (a vector) is given by $x \hat{x}$, it follows that the acceleration (a vector) is proportional to the displacement and that the acceleration and the displacement are always in opposite directions. The following examples illustrate this result.

If $\theta = 0$, the situation is as follows:



If $\theta = 180^\circ$, the directions of the vectors are as follows:



What is the situation if $\theta = 90^\circ$ or 270° ?

If we look again at the definition of simple harmonic motion in § 8.1, we arrive at the conclusion that the particle at Q executes such a motion.

Other characteristics of the motion of Q (SHM) can now be found. The distance (magnitude of the displacement) of Q from the centre O at any time t is given by

$$x = r \cos \theta.$$

Since $\theta = \omega t$ if t is measured from the instant when P and Q are at A ($\theta = 0$ if $t = 0$),

$$x = r \cos \omega t. \quad (8.4)$$

The velocity of Q is given by the horizontal component of $\vec{v}_p$;

$$\begin{aligned} \vec{v} &= -v_p \sin \theta \hat{x} \\ &= -\omega r \sin \omega t \hat{x}. \end{aligned} \quad (8.5)$$

The period T for a complete oscillation is the same as the time taken by P to describe the circle once. Thus

$$\begin{aligned} T &= 2\pi/v_p \\ &= 2\pi/\omega. \end{aligned} \quad \begin{array}{l} \text{S. 9} \\ \text{with eq. } T = 2\pi\sqrt{\frac{m}{k}} \end{array} \quad (8.6)$$

8.3 Experiment 3(a) – simple harmonic motion of a mass hanging on an idealized spring

By an idealized spring we mean one for which it may be assumed that the mass of the spring is negligible, that there are no forces tending to decrease the motion of the spring (dissipative forces) and that the restoring force exerted by the spring is strictly proportional to the extension or contraction of the spring. These assumptions are never completely justified in practice, but for a light spring in which the extensions are kept small, the errors introduced are small. By a light spring we mean one whose mass is small compared to the mass hung on its lower end. A small oscillation is one that is small compared to the length of the spring.

Consider a spring (figure 8.2(a)) with natural length ℓ , with its upper end fixed. If an object with mass m is attached to the spring (figure 8.2(b)), it will reach equilibrium with the spring extended by $\Delta\ell$. Now the upward force exerted by the spring on the object is equal to the weight of the object. Since

$$\vec{P} = k\Delta\ell \hat{x}$$

where k is the spring constant, it follows that

$$\vec{P} - mg \hat{x} = 0. \quad (8.7)$$

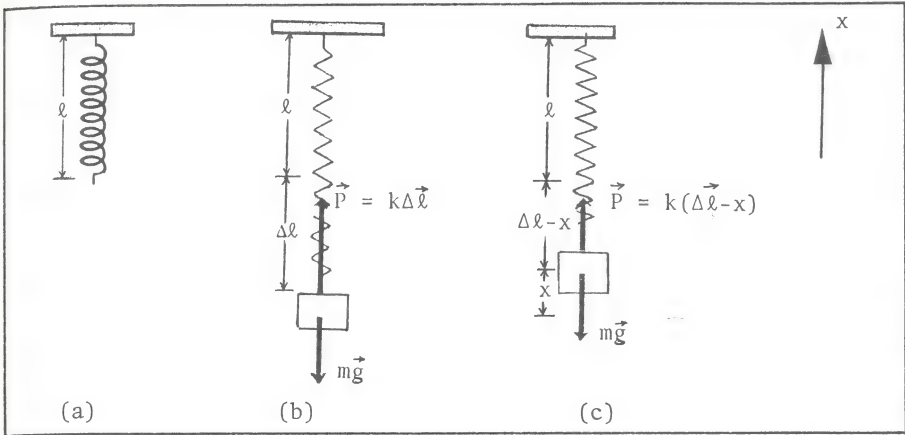


Figure 8.2 Spring with natural length ℓ .

Suppose the object is a distance x about the equilibrium position (Figure 8.2(c)). The elongation of the spring is now $\Delta\ell - x$, the upward force on the object is $k(\Delta\ell - x) \hat{x}$ and the resultant force on the object is therefore

$$\vec{F} = k(\Delta\ell - x) \hat{x} - mg \hat{x}$$

$$= -kx \hat{x}$$

or

$$\vec{a} = -(k/m)x \hat{x}. \quad (8.8)$$

The resulting force is therefore proportional to the displacement of the object *from the equilibrium position*. If it is brought into motion, it will oscillate with an angular frequency

$$\omega = (k/m)^{\frac{1}{2}}. \quad (8.9)$$

The acceleration is proportional to and in the opposite direction from the displacement.

The experiment includes the following:

1. A number of known masses are supplied. Determine the elongation of the spring Δl for each mass.
2. Determine the spring constant from the slope of a suitable graph. (Make sure that the sign of the slope of the graph is correct).
3. Determine the period of oscillation of the spring-mass system for each of the known masses.
4. Draw a suitable straight line graph and determine the spring constant from the slope of this graph.
5. Compare the values of k obtained from (2) and (4) and draw a conclusion.
6. Do a linear regression analysis and determine s_k .

8.4 Experiment 3(b) – the simple pendulum

A simple pendulum is defined as a particle of mass m suspended from a point C by a string of length ℓ and of negligible mass (figure 8.3). If the particle is pulled aside to a position A so that the string makes an angle θ with the vertical OC and is then released, the pendulum will oscillate between A and the symmetric position B .

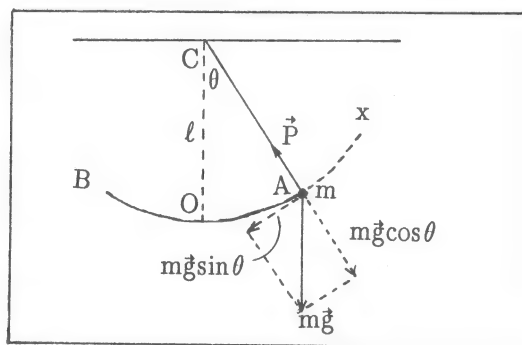


Figure 8.3 Simple pendulum.

Consider a rectangular system of axes as indicated in figure 8.3. Note that the particle, mass m , is at the origin of the system of axes. The system of axes rotates with the particle.

The forces acting on the particle when the string makes an angle θ with the vertical are $\vec{P}$ exerted by the string on the particle and $m\vec{g}$ exerted by the earth on the particle. The force $m\vec{g}$ can be resolved into two components : $-mg\cos\theta\hat{y}$ parallel to the string and a component $-mg\sin\theta\hat{x}$ along the direction of instantaneous motion of the particle, i.e. perpendicular to the string. The component $-mg\sin\theta\hat{x}$ produces the *tangential acceleration* $\vec{a}_t$ of the mass. Therefore

$$m\vec{a}_t = -mg\sin\theta\hat{x}$$

or

$$\vec{a}_t = -g\sin\theta\hat{x}.$$

If θ is small and measured in radians, $\sin\theta \simeq \theta$ and

$$\vec{a}_t \simeq -g\theta\hat{x}. \quad (8.10)$$

In terms of the arc length $OA = s = \ell\theta$, the expression for the tangential acceleration becomes

$$\vec{a}_t \simeq -\frac{gs}{\ell}\hat{x}. \quad (8.11)$$

The acceleration of the particle along the circle is therefore directly proportional to the displacement along the circle measured from O. The motion is therefore approximately simple harmonic about O as centre with the constant $\omega = (g/\ell)^{\frac{1}{2}}$. The period of the motion is

$$T = \frac{2\pi}{\omega}$$

$$\begin{aligned}
 &= \frac{2\pi}{(g/\ell)^{\frac{1}{2}}} \\
 &= 2\pi\left(\frac{\ell}{g}\right)^{\frac{1}{2}}.
 \end{aligned}
 \tag{8.12}$$

It should be pointed out that the particle also experiences a centripetal acceleration $\vec{a}_c$, given by

$$\vec{a}_c = \frac{v^2}{\ell} \hat{y} \tag{8.13}$$

which is directed towards the centre of the circular motion, i.e. the point C in figure 8.3. Reference to the figure shows that the force that produces the centripetal acceleration is $(\vec{P} - mg\cos\theta \hat{y})$. Hence the application of Newton's second law yields

$$\begin{aligned}
 ma_c &= mv^2/\ell \\
 &= P - mg\cos\theta.
 \end{aligned}
 \tag{8.14}$$

The experiment includes the following:

1. Determine the period of the pendulum as a function of its length. You have to decide how many oscillations are required for each length of the pendulum.
2. Draw a suitable straight line graph.
3. Determine g and s_g by doing a linear regression analysis.

8.5 Questions

- 8.1 A student wants to determine the dependence of T on ℓ in the formula for the period of a simple pendulum. He assumes that

$$T = k\ell^n$$

and measured T for different lengths of the pendulum. Describe the graph the student should draw to find n .

- 8.2 A mass m is suspended from a spring and causes an elongation x . An identical spring is connected in series with it and the mass m suspended from the combination. What is the elongation of the combination?
- 8.3 Answer the following questions with reference to the simple pendulum:
- a) Where is the speed a maximum?
 - b) Where is the acceleration a maximum?
 - c) Where is the kinetic energy a maximum? What is the value of the potential energy at this position in terms of the maximum speed?
 - d) Is the sum of the potential and kinetic energies a constant?
 - e) Why does a simple pendulum not keep on oscillating ad infinitum?
- 8.4 Use a figure to show that the displacement and acceleration of the mass in Experiment 3(a) are always in opposite directions.
- 8.5 Estimate the error in g if the angle θ in experiment 3(b) is 30° .
- 8.6 A mass on the end of a string is swinging back and forth as a pendulum. Is it in equilibrium at any point of the motion?
- 8.7 A 2 kg mass is in static equilibrium, hanging at the bottom end of a long vertical spring. When pulled down 0.4 m below the equilibrium position and released, it vibrates with a period of 2 s. What force does the spring exert on the mass when the mass is

- a) at its lowest point;
 - b) passing through the midpoint of its motion;
 - c) at its highest point?
- 8.8 A light spring is hanging from the ceiling. A load is attached to the free end of the spring and it is observed that the load, when released, moves downward a distance of 0.5 m before starting to rise again. What is the period of this SHM?
- 8.9 Show that when only small oscillations of a simple pendulum are considered, the centripetal acceleration is small, i.e. the magnitude of the tension in the string is approximately mg .
- 8.10 What would the shape of the curve drawn to obtain the force constant of the spring be if we continue to increase the load?
- 8.11 If a body is placed in the scale pan, its height above the ground is decreased. What is the effect on its gravitational potential energy. How is the energy conserved in this case?
- 8.12 A body of mass 0.1 kg hangs from a long spiral spring. When pulled down 0.1 m below its equilibrium position and released, it vibrates with a period of 2 s.
- a) What is its velocity as it passes through the equilibrium position?
 - b) What is its acceleration when it is 0.05 m above the equilibrium position?
 - c) When it is moving upward, how long a time is required to move from a point 0.05 m below its equilibrium position to a point 0.05 m above it?
 - d) How much will the spring shorten if the body is removed?

- 8.13 A block suspended from a spring vibrates with SHM. At an instant when the displacement of the block is equal to half the amplitude, what fraction of the total energy of the system is kinetic and what fraction is potential?
- 8.14 A pendulum clock which keeps time correctly at a point where $g = 9.8 \text{ m s}^{-2}$, is found to lose 10 s per day at a higher altitude. Find the value of g at the new location.
- 8.15 It is often said that the period of a pendulum depends on the mass of the bob. Criticise this statement.
- 8.16 What is the length of a second pendulum ($T = 2 \text{ s}$)?
- 8.17 Why is it necessary that the amplitude of swing used for each length be always less than one tenth of the length being used?
- 8.18 What are four possible sources of systematic error in the pendulum experiment?
- 8.19 At what point of its swing has the pendulum bob its greatest velocity? Explain.
- 8.20 At what point of its swing has the pendulum bob its greatest acceleration? Explain.
- 8.21 The following results were collected by a student who executed the simple pendulum experiment:

Radius of bob (cm):	0.619	0.617	0.612	0.615	0.620
Length of string (m):	0.510	0.730	1.014	1.221	1.505
Time for 20 oscillations (s):	28.3	34.8	40.0	45.0	49.5

Execute steps (1), (2) and (3) as specified in § 8.4.

8.22 The following results were collected by a student who executed the simple pendulum experiment:

Radius of bob (cm):	0.71	0.70	0.69	0.71	0.70	0.70
Length of string (m):	10.0	15.0	21.0;	33.0	46.3	63.2
Time for 20 oscillations (s):	310.5	390.0	455.3	580.5	695.1	790.3

Execute steps (1), (2) and (3) as specified in § 8.4.

8.23 The following results were collected by a student who executed a simple pendulum experiment:

Radius of bob (cm):	0.31	0.30	0.29	0.32	0.31	0.30
Length of string (m):	5.3	10.8	16.3	28.3	38.2	55.0
Time for 20 oscillations (s):	45.2	66.3	83.2	112.1	122.4	150.2

Execute steps (1), (2) and (3) as specified in § 8.4.

Experiments in electromagnetism

Chapter 9

Electrical measurements (exp. 4)

The beauty of electricity is not that the power is mysterious and unexpected, but that it is under law and that the taught intellect can now even govern it largely.

Michael Faraday (1791–1867)

9.1	Introduction	188
9.2	Experiment 4(a) - Ohmic behaviour : resistor	188
9.3	Experiment 4(b) - non-ohmic behaviour : diode	191
9.4	Experiment 4(c) - the potentiometer	193
9.5	Experiment 4(d) - the Wheatstone bridge	195
9.6	Resistance of a colour coded resistor	196
9.7	Experiment 4(e) - construction of an ammeter	198
9.8	Experiment 4(f) - Construction of a voltmeter	199
9.9	Questions	200

9.1 Introduction

The experiments described in this chapter are based on the measurement of electrical currents, potential differences and resistances. It is therefore essential to understand why electrical current can flow in a conductor and why the conductor offers resistance to the flow of current. The difference between the behaviour of ohmic and non-ohmic devices is illustrated in §9.2 and 9.3

9.2 Experiment 4(a) – Ohmic behaviour : resistor

In a metal the valence electrons are not bound to individual atoms but are free to move within the lattice of metal atoms. These electrons are called the conduction electrons. In copper there is one such electron per atom, the other 28 are bound to the nucleus.

Although the individual conduction electrons in a metal move at quite high velocities (about 10^6 m s^{-1}), this motion in the absence of an externally imposed electric field is completely random and results in no net flow of charge. We may think of this random motion as resulting from random elastic collisions between the electrons and the metal lattice. The average time between collisions is known as the *mean free time* and the average distance travelled between collisions is called the *mean free path*. This is the same terminology used in discussing the random motion of molecules in a gas. Because of the randomness of this motion, on the average no net charge is transferred. If an electric field is applied to the metal, the path of each electron is bent in its random motion in the direction of the

force produced by the field. Figure 9.1 is a crude picture of the effect of the external field in causing the electron paths to curve in the direction of the force to produce the net *drift velocity*.

This curvature results from the acceleration of the charge between collisions. We may assume that on each collision with the lattice, the excess energy picked up by the electron in the external fields is lost to the lattice. We now can associate the term mean drift velocity $\vec{v}$ with the acceleration due to the external field. During each free path the electric field produces a uniform acceleration $\vec{a} = e\vec{E}/m$. Each collision reduces the accumulated velocity produced by the external field to zero, after which the acceleration again causes a uniform velocity increase until the next collision with the lattice. The drift velocity is the time average of this motion superposed on the generally much higher random velocity of the electrons.

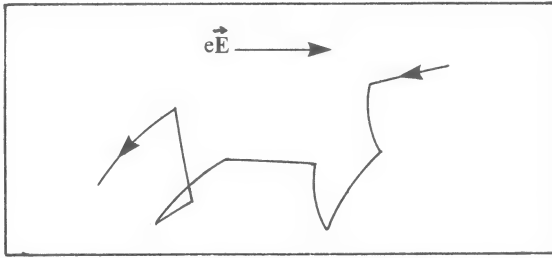


Figure 9.1 Drift of electrons in an electric field.

Since the drift velocity is always much less than the random velocity of the electrons, the mean time between collisions is independent of the applied field. Thus the only factor influencing the drift velocity is the acceleration due to the electric force $e\vec{E}$ on the electrons. Since the current I is proportional to the drift velocity, we have

$$I \propto E \quad (9.1)$$

which is the result needed to understand Ohm's law.

For the simple case of a conductor of uniform cross section and length L , an applied potential difference V between the ends gives rise to a uniform field given by

$$E = V/L. \quad (9.2)$$

For the conductor we may replace the field in the proportionality equation (9.1) by V/L to get

$$I \propto V \quad (9.3)$$

or

$$V = IR \quad (9.4)$$

where R is the resistance of the conductor. This is the general form of Ohm's law where R is a constant.

The experiment includes the following:

1. Construct a circuit as shown in figure 9.2.

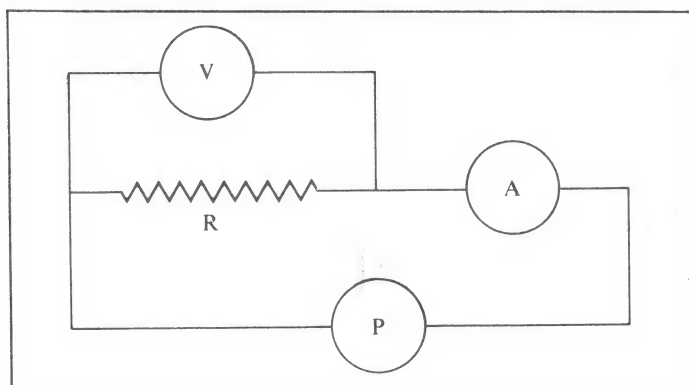


Figure 9.2 Circuit diagram for experiment 4(a).

V, A and P are a voltmeter, an ammeter and a power supply respectively. R is the unknown resistor.

2. Measure the current flowing through the resistor as a function of the potential difference across the resistor.
3. Plot a suitable straight line graph. Do a linear regression analysis of the data and obtain the resistance and the standard error on the resistance from the graph.



9.3 Experiment 4(b) – non-ohmic behaviour : diode

Silicon and germanium have four electrons in their outermost sub-shells and crystallise in the diamond structure. Each atom is surrounded by four nearest neighbours with which it forms covalent bonds. A small amount of energy is required to break some of the bonds and to set an electron free. As the temperature is increased, the number of free electrons, which could result in electrical conduction, is increased. The removal of an electron from a covalent bond leaves a positively charged vacancy. An electron from a neighbouring atom can drop into the vacancy. In this way the vacancy, called a *hole*, can move through the lattice, serving as an additional current carrying mechanism, known as *intrinsic conductivity*.

If a small amount of arsenic, having five electrons in the outer sub shell, is mixed with germanium, the arsenic atom can replace a germanium atom because these atoms have approximately the same size. Four of the valence electrons of the arsenic atom become involved in the covalent bonds with neighbours while the fifth electron remains very loosely bound to the arsenic atom. This electron can escape very easily and contribute to electrical conduction. This mechanism is by far the dominant conduction mechanism. Such a material in which the conduction is almost entirely due to negative charge motion, is called an *n-type semiconductor*.

If an element such as gallium with only three valence electrons is added, it uses one of the electrons of a neighbouring germanium atom to form the four covalent bonds, leaving the germanium with the hole. In this case the hole can move through the lattice. Such a material is called a *p-type semiconductor*.

If a crystal of silicon or germanium is grown such that one region is an n-type semiconductor and an adjacent region a p-type semiconductor, a *diode* is obtained. If a potential difference is applied across the diode, conduction takes place much more readily in the direction $p \rightarrow n$ than in the reverse direction. When the p region is at a higher potential than the n region, holes in the p region flow into the n region. Both regions contribute substantially to the current. When the polarity is reversed, the resulting electric field tends to push electrons from p to n and holes from n to p. However, there are very few electrons in the p region. A similar situation prevails in the n region and therefore the current is smaller than with the opposite polarity.

It can be shown that the relationship between the current and the voltage across a diode is given by

$$I = I_0(e^{AV} - 1), \quad (9.5)$$

where $A = e/(\eta kT)$. T , e and k are the absolute temperature, electronic charge and the Boltzman constant respectively. I_0 is the reverse saturation current and $\eta = 1$ for Ge and 2 for Si.

Note that if $AV \ll 0$, equation (9.5) can be written as

$$I \approx I_0(-1) = -I_0. \quad (9.6)$$

Therefore, equation (9.5) can be rewritten as

$$I/I_0 = e^{AV} - 1. \quad (9.7)$$

The experiment includes the following:

1. Construct a circuit similar to the one shown in figure 9.2 with the resistor replaced by the diode.
2. Measure the current flowing through the diode as a function of potential difference across the diode.
3. Reverse the diode in the circuit and repeat (2).
3. Plot a graph of I versus V .
4. Obtain I_0 from the measurements.
6. Draw a suitable straight line graph of equation (9.7), and determine A and s_A . You have to do a linear regression analysis.
7. Calculate the absolute temperature from the value of A and compare it with the temperature at which the experiment was executed (~ 295 K).

9.4 Experiment 4(c) – the potentiometer

A potentiometer is a voltage comparison device that uses a galvanometer as a null indicator. The circuit is shown in figure 9.3. It consists of a voltage source E connected to a uniform resistance wire CD , length ℓ . The potential difference across AB is adjusted by moving the sliding contact K along CD . If a known voltage source E_s is connected across AB (note the polarities of E and E_s), the current through the galvanometer will be zero if

$$E_s = I_0 R_s,$$

where R_s is the resistance of the wire between D and K_s . If the known voltage source is *replaced* by the unknown voltage source E_x the galvanometer will show no deflection if

$$E_x = I_o R_x,$$

where R_x is the resistance of the wire between D and K. From these two equations it follows that

$$E_x/E_s = R_x/R_s. \quad (9.8)$$

Since the resistance wire is uniform, the resistances are proportional to the lengths of the wires between D and K_s , and D and K_x .

Therefore

$$E_x/E_s = \ell_x/\ell_s \quad (9.9)$$

where ℓ_s is the measured length between D and K_s and ℓ_x that between D and K_x .

Since no current flows through the unknown voltage source, its emf is determined.

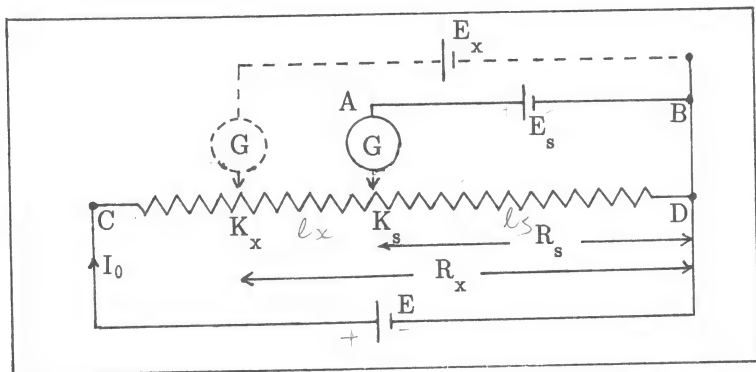


Figure 9.3 Potentiometer

The experiment includes the following:

1. You are supplied with 3 voltage sources: a variable source E (see figure 9.3), a torch cell E_s and a black box E_x .
2. Determine ℓ_s for zero reading of the galvanometer with E_s connected across AB.
3. Replace E_s with E_x and determine ℓ_x .
4. Calculate E_x .

9.5 Experiment 4(d) – The Wheatstone bridge

A Wheatstone bridge is widely used for the precise measurement of resistance. The circuit is shown in figure 9.4.

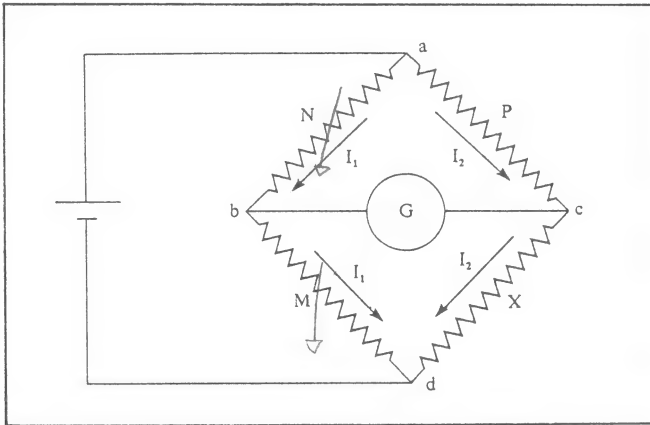


Figure 9.4 Wheatstone bridge.

M and N are adjustable resistors, P is a known resistance and X is the unknown resistance. The values of M and N are adjusted until the galvanometer G shows no deflection. Points b and c are then at the same

potential, i.e. the potential drop from a to b equals that from a to c. Similarly, the drop from b to d equals that from c to d. Since the galvanometer current is zero, the current I_1 in M, equals that in N and the current I_2 in P equals that in X. Since $V_{ab} = V_{ac}$, it follows that

$$I_1 N = I_2 P$$

and since $V_{bd} = V_{cd}$,

$$I_1 M = I_2 X.$$

Therefore

$$N/M = P/X$$

or

$$X = MP/N.$$

(9.10)

The experiment includes the following:

1. Construct the circuit shown in figure 9.4. Use the resistance wire of the potentiometer (Experiment 4(c)) for resistors N and M. The sliding contact is represented by b in figure 9.4.
2. Determine the ratio M/N .
3. Calculate X.

9.6 Resistance of a colour coded resistor

Normally the resistance of a resistor is indicated by four colour bands on the resistor as illustrated in figure 9.5.

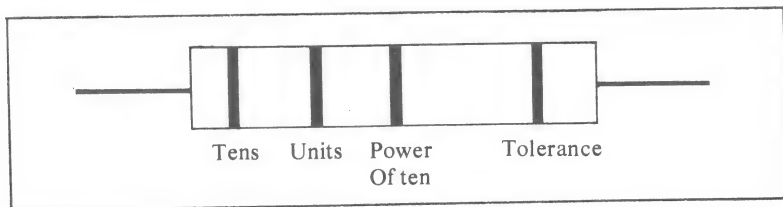


Figure 9.5 Colour coded resistor.

The numbers associated with the different colours are listed in table 9.1.

Table 9.1 Numbers for colour coded resistors.

Band	Colour	Number
First two bands	Black	0
	Brown	1
	Red	2
	Orange	3
	Yellow	4
	Green	5
	Blue	6
	Violet	7
	Grey	8
	White	9
Third band	Gold	$\times 10^{-1}$
	Black	$\times 10^0$
	Brown	$\times 10^1$
	Red	$\times 10^2$
	Orange	$\times 10^3$
	Yellow	$\times 10^4$
	Green	$\times 10^5$
	Blue	$\times 10^6$
	Violet	$\times 10^7$
	Grey	$\times 10^8$
	White	$\times 10^9$
Fourth band	Silver	$\pm 10\%$
	Gold	$\pm 5\%$

The combination of colours red, green, orange and silver therefore indicates a resistance of $(25 \times 10^3 \pm 10\%)$ ohm, and a resistor marked red, red, gold and gold will have a resistance $(22 \times 10^{-1} \pm 5\%)$ ohm.

9.7 Experiment 4(e) – Construction of an ammeter

The construction of an ammeter is described in Chapter 5.

The experiment includes the following:

1. A number of resistors are supplied in a container. Draw up a list of the colour codes and resistance values of *each* of these resistors.
2. Select *one* of these resistors suitable to be used as shunt R_s and construct the circuit as illustrated in figure 9.6 where A represents a multimeter set to measure current. *Before* you switch on the power supply P, ask the instructor to check the circuit.

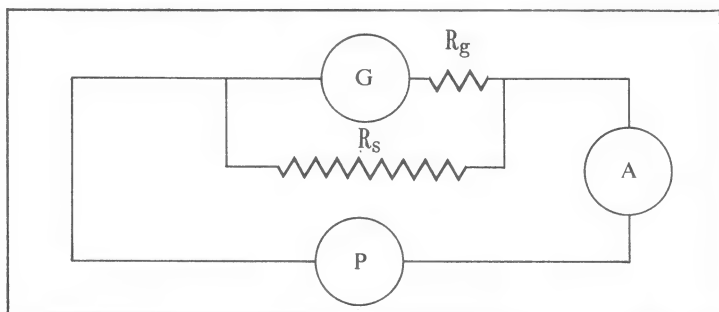


Figure 9.6 Construction of an ammeter.

3. The scale on the galvanometer G has divisions marked from 0 to 40. Increase the current until the reading on the galvanometer is 5 divisions. Read the current on the multimeter A.
4. Repeat at 10, 15, 20, 25, 30, 35 and 40 divisions and draw up a calibration table.
5. Draw a calibration graph.
6. Calculate the currents through the galvanometer and the shunt R_s when the galvanometer shows 40 divisions.

9.8 Experiment 4(f) – Construction of a voltmeter

The construction of a voltmeter is described in Chapter 5.

The experiment includes the following:

1. A number of resistors are supplied in a container. Draw up a list of the colour codes and resistance values of *each* of the resistors.
2. Select *one* of these resistors suitable to be used as series resistor R and construct the circuit as illustrated in figure 9.7, where V represent a multimeter, set to measure voltage. Before you switch on the power supply P , ask the instructor to check the circuit.

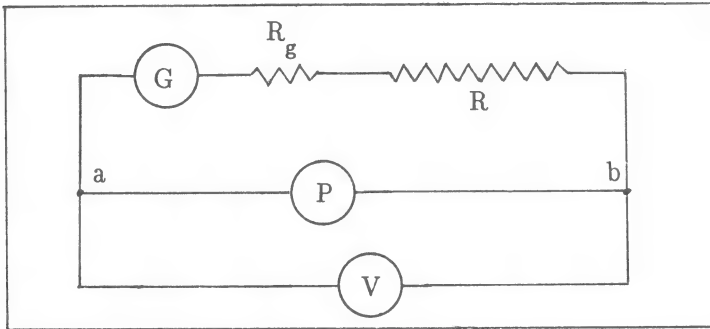


Figure 9.7 Construction of a voltmeter.

3. The scale on the galvanometer G has divisions marked from 0 to 40. Increase the current until the reading on the galvanometer is 5 divisions. Read the voltage on the multimeter V .
4. Repeat at 10, 15, 20, 25, 30, 35 and 40 divisions and draw up a calibration table.
5. Draw a calibration graph.
6. Calculate the currents through the galvanometer and the shunt R_g when the galvanometer shows 40 divisions.

9.9 Questions

- 9.1 Compare electrical conductivity with thermal conductivity.
- 9.2 What is meant by linear and non-linear circuit elements?
- 9.3 How does the resistance of a wire depend on the length, area, diameter and temperature?
- 9.4 Suppose the behaviour of a diode is given by

$$I = Ae^{(BV+C)}$$

where A , B and C are constants. V is the potential difference across the diode and I is the current flowing through it. Measured values of I and V are given below.

V	I	V^2	I^2	$\ln V$	$\ln I$
0	9.0	0	81	—	2.19
10	40.2	100	1613	2.30	3.69
20	180.0	400	32412	3.00	5.19
30	806.9	900	651000	3.40	6.69
40	3616.1	1600	13076000	3.69	8.19

The potential difference is measured in mV and the current in mA.

- What are the units of A ?
- Rewrite the equation to obtain a straight line graph.
- Draw a graph to obtain the value of B .

- 9.5 Is the direction of electron drift (not conventional current direction) from high to low potential or from low to high potential? Explain.
- 9.6 If the current in a wire is 7 A, what is the charge in C which drifts past a given point in the wire in 1 s?
- 9.7 What is the definition of the potential difference between two points in space?
- 9.8 Why do we use a potentiometer rather than a voltmeter for the measurement of an emf?
- 9.9 Do temperature changes on the slide wire affect the results obtained from the potentiometer? Explain.
- 9.10 How do the diameter and the material of the slide wire affect the results obtained by a potentiometer? Explain.
- 9.11 Draw a circuit diagram and show in which parts a current is flowing when the potentiometer is balanced.
- 9.12 What effect would a slide wire ten times the length have on the accuracy of the measurements?
- 9.13 How can a potentiometer be made to read directly in volts?
- 9.14 How does the emf of the power supply affect the results obtained by the potentiometer? Should the power supply be larger, smaller or the same as the unknown emf?
- 9.15 Discuss the importance of the correct polarity of the batteries used in the potentiometer experiment.

- 9.16 If a Wheatstone bridge is balanced, will interchanging the battery and galvanometer affect this balance? Explain. If the bridge is balanced, what would be the effect of removing the battery completely from the circuit?
- 9.17 Is the polarity of the power supply in the Wheatstone bridge of any importance?
- 9.18 An ammeter is a low resistance instrument connected in series, while a voltmeter is a high resistance instrument connected in parallel. Which instrument would be damaged when improperly connected? Explain.
- 9.19 The emf of a "dead" battery is measured by a potentiometer to be 1.5 volts. When a voltmeter of resistance $R_v = 900$ ohm is connected across the terminals of the battery, it reads 1.35 volts. What is the internal resistance of the battery?
- 9.20 Why is it necessary to wait a few seconds before taking any measurements if a light bulb is used in the Ohm's law experiment?
- 9.21 The following data were obtained for Experiment 4(a):

V (volt)	I (A)
1	0.11
2	0.19
3	0.33
4	0.41
5	0.47
6	0.59
7	0.73
8	0.80
9	0.93
10	0.99

Analyse the data as prescribed in § 9.2.

- 9.22 The following data were obtained for Experiment 4(b) with a silicon diode:

V (volt)	I (A)
0.30	75
0.25	28
0.20	10.6
0.15	3.3
0.10	1.2
0.05	0.36
0.0	0.01
-0.05	-0.13
-0.10	-0.15
-0.15	-0.19
-0.20	-0.19
-0.25	-0.20
-0.30	-0.20
-0.40	-0.20

Analyse the data as prescribed in § 9.3.

Chapter 10

Charged particle in uniform magnetic field (exp. 5)

What led me to the special theory of relativity was the conviction that the electromotive force acting on a body in motion in a magnetic field was nothing else but an electric field.

Albert Einstein (1879–1955)

10.1	Introduction	205
10.2	Thermionic emission	205
10.3	Acceleration of electrons	206
10.4	Excitation of a hydrogen atom	207
10.5	The magnetic field of a pair of Helmholtz coils	208
10.6	Motion of charged particle in uniform magnetic field	210
10.7	Apparatus	212
10.8	Experiment 5(a) - circular path of electrons	214
10.9	Experiment 5(b) - e/m of an electron	215
10.10	Questions	216

10.1 Introduction

In the experiments described in this chapter, the *motion* of a *charged particle* (an electron) in a *uniform magnetic field* will be observed and investigated. Before the experiment can be executed, the following physical aspects must be understood:

1. The liberation of electrons from a metal.
2. The acceleration of electrons in a uniform electric field.
3. The excitation of gas molecules by collisions with fast moving electrons.
4. The generation of a uniform magnetic field.
5. The force exerted on a moving electron in a uniform magnetic field and the path followed by such an electron.

10.2 Thermionic emission

If a metal is heated, the energy of the electrons inside the metal is increased. If the energy is high enough an electron may overcome the forces keeping it inside the metal and escape from it. This is called *thermionic emission*. Normally the space around a heated metal contains many electrons.

10.3 Acceleration of electrons

If a potential difference is applied across two plates as shown in figure 10.1 a uniform electric field is generated in the space between the plates.

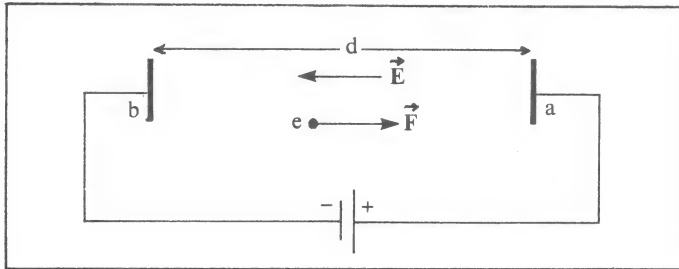


Figure 10.1 Electric field between two plates a and b.

The magnitude of the electric field is given by

$$E = V_{ab}/d \quad (10.1)$$

and is directed from the higher (anode) to the lower potential (cathode), i.e. from a to b. According to the definition of electric field intensity,

$$\vec{F} = e\vec{E} . \quad (10.2)$$

Since e , the charge of the electron, is negative, $\vec{F}$ and $\vec{E}$ are in opposite directions as shown in figure 10.1. This is the only force exerted on the electron (the gravitational force is negligible in comparison with the electrical force), and it will therefore be accelerated in the direction of $\vec{F}$. From Newton's second law

$$\begin{aligned} \vec{F} &= m\vec{a} \\ &= e\vec{E} . \end{aligned} \quad (10.3)$$

The magnitude of the acceleration is therefore

$$a = \frac{eE}{m}$$

$$= \frac{eV_{ab}}{md} \quad (10.4)$$

and it has the same direction as the force, from b to a. If an electron starts from rest at b, the magnitude of its velocity $\vec{v}$ at a is given by

$$v^2 = 2ad$$

$$= 2eV_{ab}/m$$

or

$$v = (2eV_{ab}/m)^{\frac{1}{2}}. \quad (10.5)$$

10.4 Excitation of a hydrogen atom

The energy in eV ($1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$) of an electron in a hydrogen atom is given by

$$E = -13.6n^2; \quad n = 1, 2, 3 \dots \quad (10.6)$$

where n is the *principal quantum number*. In an unexcited hydrogen atom the electron is in its lowest energy state and $n = 1$. Such an electron is stable and orbits without emitting or absorbing energy. Excitement of the electron takes place when it absorbs energy, for example if a free electron which has been accelerated by an electric field, as discussed in §10.3, hits it. If the electron absorbs less than 13.6 eV, it may absorb only just enough energy to leave the ground state and to occupy one of the permitted energy levels. This energy is numerically equal to the *excitation potential* of that

state. The excited electron next returns to its normal state. If it was excited to $n = 4$, it may jump from 4 to 1 in one step or it may go 4, 2, 1 or 4, 3, 1 or 4, 3, 2, 1. In each step of the return trip the electron must lose an amount of energy equal to the difference of the energy levels. The only mechanism available for this energy loss is through the emission of *electromagnetic radiation*.

10.5 The magnetic field of a pair of Helmholtz coils

A pair of Helmholtz coils is formed by two identical coils on the same axis having N turns each. The radius of each coil is R which is also the separation between the two coils as shown in figure 10.2.

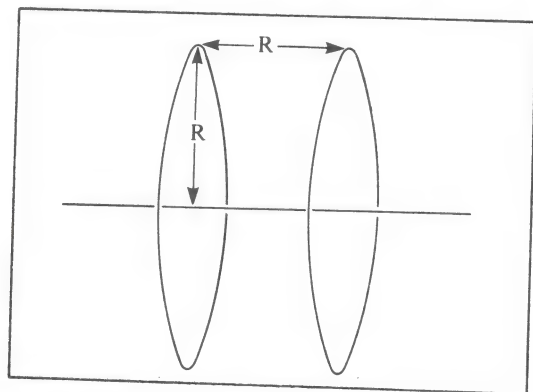


Figure 10.2 Helmholtz coils.

To calculate the magnetic field a distance from the circular current loop (see figure 10.3), we calculate the contribution $\Delta \vec{B}$ at P due to the current element $\Delta \vec{L}$.

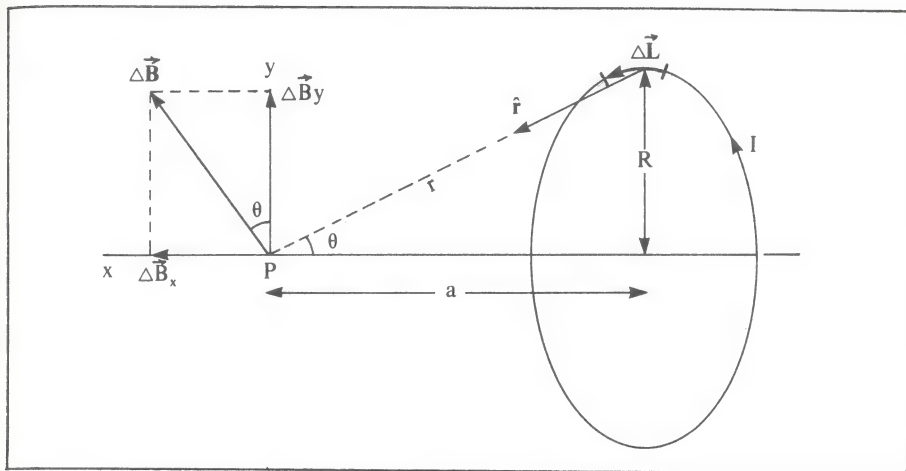


Figure 10.3 Magnetic field at a distance a from a coil.

For any one segment $\Delta \vec{L}$ the distance to P is r . Since $\Delta \vec{L}$ is perpendicular to the unit vector $\hat{r}$,

$$\begin{aligned} |\Delta \vec{B}| &= kI |\Delta \vec{L} \times \hat{r}| / r^2 \\ &= kI \Delta L / r^2. \end{aligned} \quad (10.7)$$

The magnitude of the component of $\Delta \vec{B}$ along the x axis is

$$\begin{aligned} \Delta B_x &= \Delta B \sin \theta \\ &= kI \Delta L \sin \theta / r^2 \end{aligned} \quad (10.8)$$

where the constant k is

$$k = 10^{-7} \text{ N A}^{-2}.$$

If the contributions due to all the small segments $\Delta \vec{L}$ are added, the y components cancel and the magnitude of the total x component is given by

$$\begin{aligned}
 B &= \Sigma \Delta B_x \\
 &= (kI \sin \theta / r^2) \Sigma \Delta L \\
 &= 2kI \pi R \sin \theta / r^2 \\
 &= 2\pi k I R^2 / r^3.
 \end{aligned} \tag{10.9}$$

If we have a coil consisting of N identical circular current loops

$$B = 2\pi N k I R^2 / r^3. \tag{10.10}$$

If two coils are arranged to form a pair of Helmholtz coils and if the current flows in the same direction in both coils, the magnitude of the total magnetic field in tesla, at the centre of the coils is given by

$$\begin{aligned}
 B &= \frac{4\pi N k I R^2}{[a^2 + R^2]^{3/2}} \quad \dots \quad a = R/2 \\
 &= \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{4\pi N k I}{R} \\
 &= (8.99 \times 10^{-7}) \frac{NI}{R}.
 \end{aligned} \tag{10.11}$$

* This arrangement gives a uniform field over a large space between the coils.

10.6 Motions of charged particle in uniform magnetic field

Let a negatively charged particle at point O in a uniform magnetic field $\vec{B}$ (out of the plane of the paper) be given a velocity $\vec{v}$ in a direction at right

angles to the field as illustrated in figure 10.4. A force

$$\vec{F} = e\vec{v} \times \vec{B} \quad (9-12)$$

is exerted on the particle in the upward direction at this point. Please note that if the right hand screw rule is applied in this case, the negative sign of the charge must be taken into account. Therefore

$$\begin{aligned} \vec{F} &= e\vec{v} \times \vec{B} \\ &= -q\vec{v} \times \vec{B} \\ &= q\vec{B} \times \vec{v} \end{aligned}$$

where $q = |e|$. The force on a positive charge is therefore in the opposite (downward) direction. Since the force is at right angles to the velocity, it will not affect the magnitude of the velocity, but will only alter its direction.

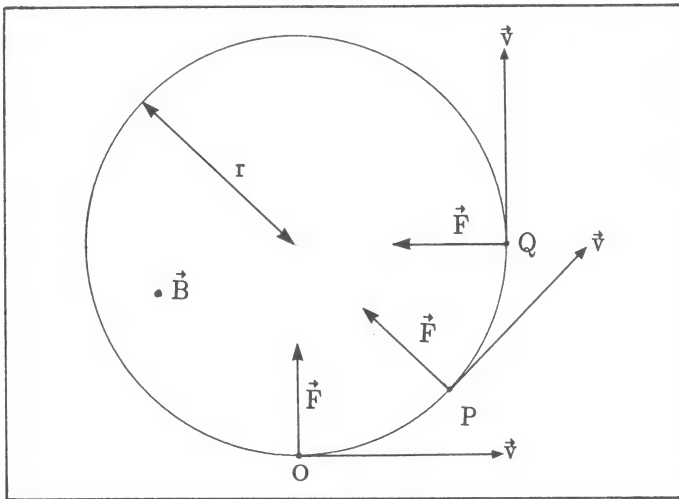


Figure 10.4 Motion of a charged particle in a uniform magnetic field.

At points such as P and Q the directions of force and velocity will have changed as shown. The magnitude of $\vec{F}$ remains constant since e and the magnitudes of $\vec{v}$ and $\vec{B}$ are constant. The particle therefore moves under the influence of a force whose magnitude is constant but whose direction is always at right angles to the instantaneous velocity of the particle. The orbit is therefore a circle with radius r and the magnitude of the centripetal force is given by

$$\begin{aligned} mv^2/r &= e|\vec{v} \times \vec{B}| \\ &= evB. \end{aligned} \quad (10.13)$$

The last step follows from the fact that $\vec{v}$ is perpendicular to $\vec{B}$. If the charged particle is accelerated from rest through a potential difference V_{ab} , its velocity is given by equation (10.5) and equation (10.13) can be rewritten as

$$\frac{e}{m} = \frac{2V_{ab}}{B^2 r^2}. \quad (10.14)$$

10.7 Apparatus

The apparatus is illustrated in figure 10.5. Free electrons are obtained from a heated filament and are accelerated by a potential difference applied across the anode and cathode A of a cathode ray tube. The tube B is filled with hydrogen gas at a very low pressure. Due to the excitation of the gas by the electrons, the path followed by the electrons is made visible. The diameter of the circular path of the electrons in the magnetic field of the Helmholtz coils C can be measured on a calibrated fluorescent screen D inside the tube.

Each of the coils has 320 turns, the radius of the coils is $R = 6.8$ cm and the half mean distance of the coils is $a = 3.4$ cm.

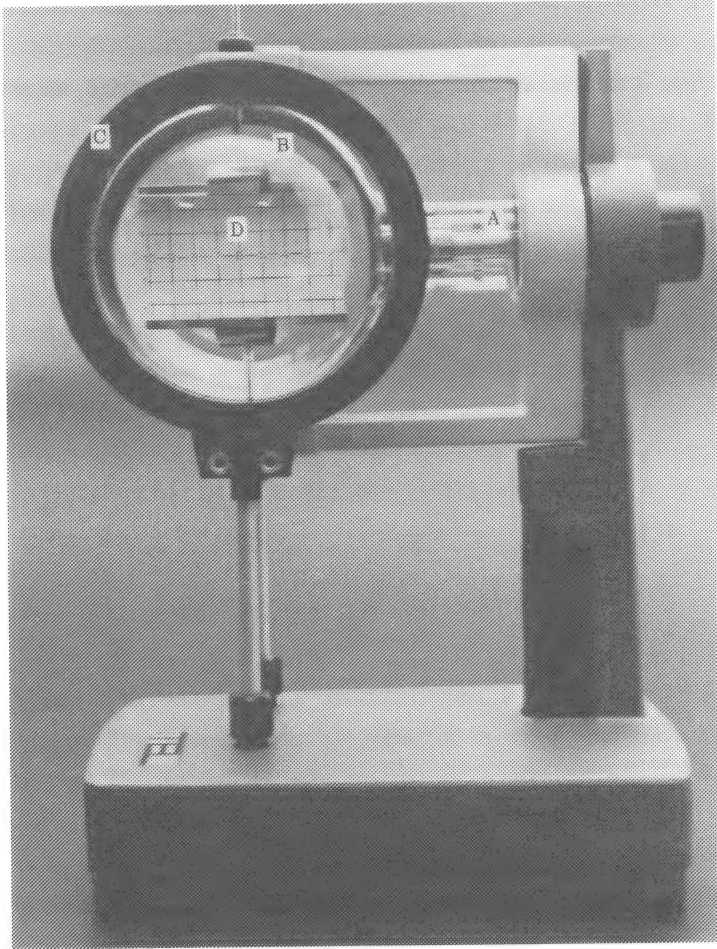


Figure 10.5 e/m apparatus.

The electron beam tube is operated with HIGH voltage sources!! Be careful not to touch any open metallic points in the circuit.!!

10.8 Experiment 5(a) - Circular path of electrons

The radius of curvature r of the electron beam visible on the fluorescent screen follows from the relation that is illustrated in figure 10.6, namely

$$r^2 = x^2 + (r - y)^2, \quad (10.15)$$

from which

$$y = \frac{1}{2r} (x^2 + y^2). \quad (10.16)$$

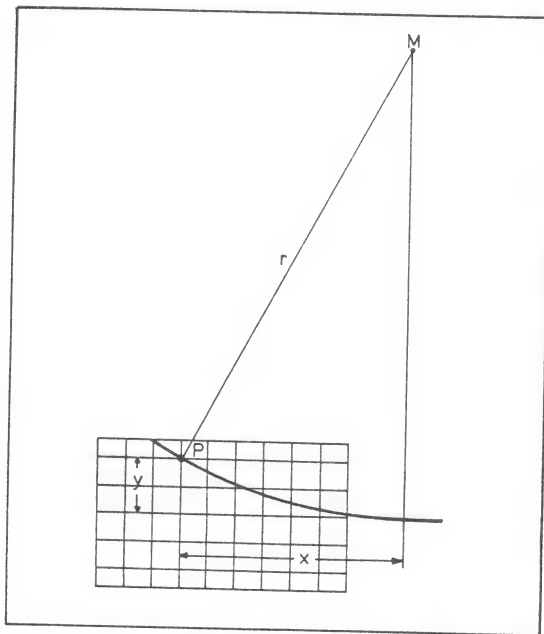


Figure 10.6 Radius of curvature of electron beam.

The experiment includes the following:

1. Switch on the accelerating potential and the filament current.
2. Increase the accelerating potential to a value between 1000 and 3000 V. Record this value.
3. Switch on the current through the Helmholtz coils. Adjust the current until the electron beam intersects more or less the fluorescent screen as illustrated in figure 10.6. Record this current.
4. Read off at least six pairs of (x; y) coordinates for this position of the beam.
5. Use equation (10.16) to draw a graph that will confirm that the electrons move in a circular path.
6. Obtain the radius of curvature r of the circular path from the graph.

10.9 Experiment 5(b) - e/m of an electron

The experiment includes the following:

1. Switch on the accelerating potential and the filament current.
2. Increase the accelerating potential to a value between 1000 and 3000 V. Record this value.
3. Switch on the current through the Helmholtz coils.
4. Adjust this current to measure the radius of curvature r of the electron beam as a function of the current. You may read only *three* (x; y) pairs to determine r for each setting of the current. At least *seven* values of the current are required.
5. Turn down the accelerating potential to zero and *then* switch off the apparatus without delay.
6. Draw a suitable straight line graph to obtain e/m . Do a linear regression analysis.
7. Determine the standard error on e/m .

10.10 Questions

- 10.1 Can you justify why it is only necessary to consider a single electron in the magnetic field rather than the many electrons which are actually present? Explain.
- 10.2 Discuss the effects of the earth's magnetic field on the results of these experiments.
- 10.3 How could one correct for the effect of the horizontal component of the earth's magnetic field?
- 10.4 What is the minimum amount of information needed to determine the direction of the magnetic force on a moving charge?
- 10.5 The equation $F = qvB\sin\theta$ involves the magnitude of three vectors $\vec{F}$, $\vec{v}$ and $\vec{B}$. Of these three vectors, which pairs are always at right angles? Which pairs may have any angle between them?
- 10.6 Explain how magnetic fields can be used to measure a charged particle's momentum.
- 10.7 A length of wire carries a steady current. It is bent first to form a circular plane coil of one turn. The same length is now bent more sharply to give a double loop of smaller radius. What is the ratio of the magnetic flux densities at the centres of the two coils?
- 10.8 Under what conditions is the motion of a charged particle in a uniform magnetic field a straight line? Is the particle accelerated?

- 10.9 A charged particle moves in a uniform magnetic field. How does the velocity of the particle vary as it moves? How does the energy vary?
- 10.10 Why should the pressure inside the cathode ray tube be low?
- 10.11 Explain why $\Sigma \Delta B_y = 0$ in figure 10.3.
- 10.12 What is the reaction to the force $\vec{F}$ in figure 10.4?
- 10.13 A charged particle moves in a uniform magnetic field. How does the energy of the particle vary as it moves? Compare your answer with the motion of a charged particle in a uniform electric field
- 10.14 Define the cross product of two vectors $\vec{A}$ and $\vec{B}$. If $\vec{C} = \vec{A} \times \vec{B}$, what is
- (a) the magnitude of $\vec{C}$ and
 - (b) the direction? What is the direction of $\vec{D}$ if $\vec{D} = -\vec{A} \times \vec{B}$?
- 10.15 Write down the vector equation for the force on a charged particle that moves with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
- 10.16 In the experiment, the charged particles are first accelerated to a velocity $\vec{v}$. Explain how this is done. Find an expression for v in terms of V , the accelerating potential.
- 10.17 How many forces act on the electron while it is moving in the circular orbit? Just give the number, 1, 2, 3, 4 or 5. Discuss the nature of each of the forces.
- 10.18 There are two circular coils to produce a homogeneous magnetic field. Why must the field be homogeneous?

- 10.19 Just how is the electron beam made visible in the e/m tube? Do not only say light is emitted due to the ionization of the particular gas in the tube by the fast moving electrons. Discuss the physical process which results in the emission of light.
- 10.20 In Experiment 5(b) I and r are variables while V is a constant. How would you plot the results to obtain a straight line graph and what is the slope of this graph in terms of e/m and V ?
- 10.21 Discuss another method for determining e/m .
- 10.22 An electron and an alpha particle (a doubly ionized helium atom) both move in circular paths in a magnetic field with the same tangential velocity. Compare the number of revolutions they make per second. The mass of the alpha particle is 6.68×10^{-27} kg.

- 10.23 The following results were obtained for Experiment 5(a):

$$V = 3000 \text{ V}$$

$$I = 0.3 \text{ A}$$

$x(\text{cm})$	: 2.0	3.0	4.0	5.0	6.0	7.0
$y(\text{cm})$	: 0.2	0.4	0.7	1.1	1.5	1.9

Analyse the results as described in §10.8.

- 10.24 The following results were obtained for Experiment 5(b) :

$I(\text{A})$	$x_1(\text{cm})$	$y_1(\text{cm})$	$x_2(\text{cm})$	$y_2(\text{cm})$	$x_3(\text{cm})$	$y_3(\text{cm})$
0.08	5.0	0.3	7.0	0.5	10.0	1.0
0.13	4.0	0.3	6.0	0.6	9.0	1.3
0.19	4.0	0.4	6.0	0.8	9.0	1.9
0.24	3.0	0.4	6.0	1.2	8.0	1.9
0.37	3.0	0.5	5.0	1.3	6.0	1.8
0.48	2.0	0.4	4.0	1.2	5.0	1.7

$V = 3000 \text{ V}$. Analyse the results as described in §10.9

Chapter 11

Magnetic force on a current-carrying conductor (exp. 6)

In every explanation of natural phenomena we are compelled to leave the sphere of sense perception and to pass to things which are not the objects of sense and are defined only by abstract conceptions.

Hermann von Helmholtz (1821–1894)

11.1	Introduction	220
11.2	Theory	220
11.3	Apparatus	221
11.4	Experiment 6(a) - Dependence of magnetic force on electric current	223
11.5	Experiment 6(b) - Dependence of magnetic force on the length of the conductor	224
11.6	Experiment 6(c) - Dependence of magnetic force on magnetic field strength	225
11.7	Questions	226

11.1 Introduction

The principles involved in this experiment are the same as those described in Chapter 10. The flow of the current in a straight wire is the movement of electrical charge but in this case the electrical charges are confined to the conductor and therefore do not describe a circular orbit as described in Chapter 10.

11.2 Theory

The force $\vec{F}$ on a charge q moving in a magnetic field $\vec{B}$ is given by

$$\vec{F} = q\vec{v} \times \vec{B} . \quad (11.1)$$

Suppose that the charge moves through a straight conductor (wire) length L . Consider a vector $\vec{L}$ parallel to the wire with magnitude equal to the length of the wire and in the same direction as the conventional current.

The displacement of the charge is

$$\vec{s} = \vec{L} \quad (11.2)$$

and therefore the velocity is

$$\vec{v} = \vec{s}/t = \vec{L}/t . \quad (11.3)$$

From equation (11.1) it follows that

$$\vec{F} = q(\vec{L} \times \vec{B})/t \quad (11.4)$$

if the field $\vec{B}$ is uniform over the length L . Since

$$I = q/t \quad (11.5)$$

it follows that

$$\vec{F} = I(\vec{L} \times \vec{B}). \quad (11.6)$$

The cross product in equation (11.6) indicates that the force $\vec{F}$ is perpendicular to both $\vec{L}$ and $\vec{B}$.

11.3 Apparatus

The apparatus is shown in figure 11.1.

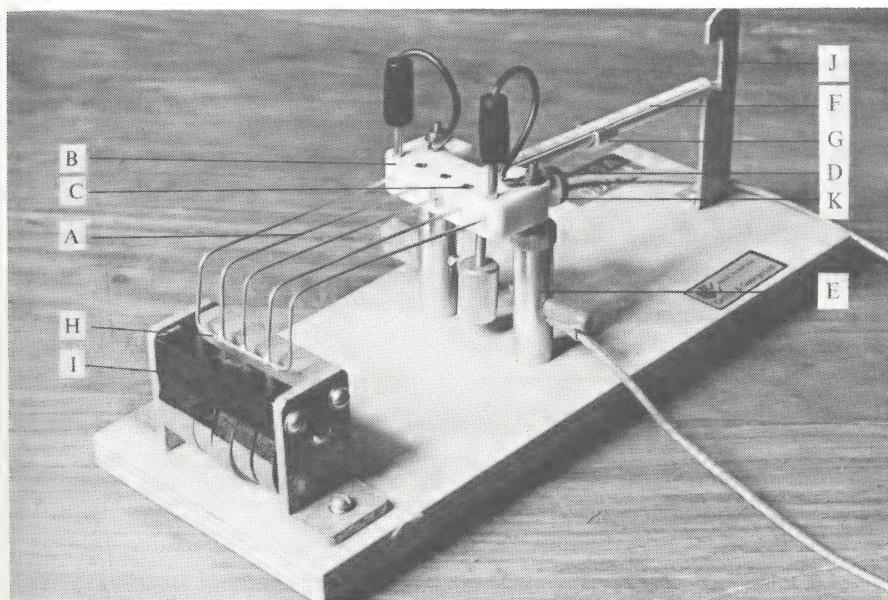


Figure 11.1 Magnetic force on a current-carrying conductor apparatus.

Five equally spaced straight wires A are attached horizontally to a non-conductor B. Also attached to B are five electrical connectors C (one for each of the five wires); two vertical metal rods D which rest on two thicker vertical metal rods E; and a horizontal aluminium rod F which serves as a centimeter scale. A counter-poise G of known mass m can slide along F. About 10 cm from B the wires A are bent vertically downwards. About 2 cm from the bend the vertical wires are connected to a piece of straight horizontal wire H. A set of six identical U-shaped permanent magnets I is arranged in such a way that the horizontal wire H is in the magnetic field and perpendicular to it. At the other end of the apparatus a reference mark J indicates the horizontal position of the aluminium rod. The output terminals of a power supply are connected to metal rods E and the circuit is completed by connecting two of the connectors C to the metal rods D. By selecting different positions of C the length L of the horizontal conductor which carries an electric current can be varied. The current can be changed by adjusting the current and/or voltage controls on the power supply. The magnitude of the magnetic field can be varied by changing the number of magnets. (Make sure that the magnetic fields of the different magnets are all in the same direction).

If the current is off and the zeroing screw K adjusted until the apparatus is horizontal with the counter-poise at the zero position (Z in Figure 10-2), the principle of moments yields

$$\vec{\Gamma}_L = \vec{\Gamma}_R \quad (11.7)$$

where the subscripts L and R refer to counter-clockwise and clockwise directions respectively.

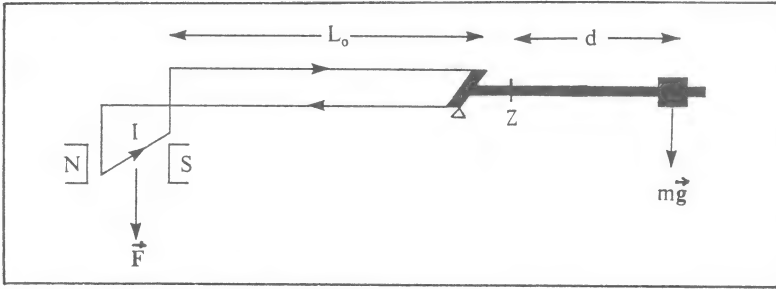


Figure 11.2 Force diagram.

If the current is switched on, the horizontal wire will experience a magnetic force of magnitude $F = ILB$ (figure 11.2). Depending on the directions of $\vec{L}$ and $\vec{B}$, $\vec{F}$ can be either upwards or downwards. If $\vec{F}$ is downwards, $\vec{\Gamma}_L > \vec{\Gamma}_R$ and the apparatus will rotate counter-clockwise. It can be aligned horizontally again by increasing $\vec{\Gamma}_R$ until $\vec{\Gamma}_L = \vec{\Gamma}_R$, i.e. the counter-poise has to be shifted towards the right. Under these circumstances the additional clockwise moment mgd is equal to the moment of the magnetic force FL_0 , i.e. the magnitude of the force is

$$\begin{aligned} F &= mgd/L_0 \\ &= ILB, \end{aligned} \quad (11.8)$$

where d is the distance measured on the scale on the aluminium rod F .

11.4 Experiment 6(a) – dependence of magnetic force on electric current

The experiment includes the following :

1. Adjust the zeroing screw until the apparatus is horizontal with $I = 0$ and the counter-poise at the zero position.

2. Switch on the power supply and increase the current slowly until the apparatus rotates anti-clockwise. Use maximum values for L and B . Adjust the counter-poise until the apparatus is horizontal.
3. Record the current and distance d .
4. Calculate the magnitude of the magnetic force in newton using the experimentally obtained value of d , the given value of m , the value of L_0 which can be measured and $g = 9.8 \text{ ms}^{-2}$.
5. Repeat steps (1) to (4) until at least six sets of readings have been obtained. *Do not exceed 2 A.*
6. Draw a suitable straight line graph to prove that $F \propto I$.
7. Do a linear regression analysis of the data.
8. Measure L and calculate B and s_B in tesla from the slope of the graph.

11.5 Experiment 6(b) – Dependence of magnetic force on the length of the conductor

The experiment includes the following:

1. Measure the total length of the horizontal wire and calculate the average length between a pair of adjacent vertical wires.
2. Adjust the zeroing screw until the apparatus is horizontal with $I = 0$ and the counter-poise in the zero position.
3. Set the current at a convenient value and keep it constant throughout the experiment. *Do not exceed 2 A.*
4. Obtain four values of d corresponding to the four different lengths L .
5. Calculate the magnitudes of $\vec{F}$ in newton using the experimentally obtained values of d , the given value of m , the value of L_0 which can be measured and $g = 9.8 \text{ m s}^{-2}$.

6. Draw a suitable straight line graph to illustrate that $F \propto L$.
7. Do a linear regression analysis of the data.
8. Calculate B and s_B in tesla from the slope of the graph.

11.6 Experiment 6(c) – Dependence of magnetic force on magnetic field strength

This experiment is executed after Experiment 6(a) or Experiment 6(b) has been executed.

The experiment includes the following:

1. Adjust the zeroing screw until the apparatus is horizontal with $I = 0$ and the counter-poise in the zero position.
2. Set the current at a convenient value and keep it constant throughout the experiment. *Do not exceed 2 A.* Use the maximum value of L .
3. Obtain six values of d corresponding to the six different values of B .
4. Calculate the magnitudes of $\vec{F}$ in newton using the experimentally obtained values of d , the given value of m , the value of L_0 which can be measured and $g = 9.8 \text{ m s}^{-2}$.
5. Draw a suitable straight line graph to illustrate that $F \propto B$.
6. Do a linear regression analysis of the data.
7. Calculate L from the slope of the graph and compare it with the value obtained from direct measurement.

11.7 Questions

- 11.1 Use the expression in equation (11.6) to derive the units of magnetic induction.
- 11.2 Illustrate graphically how the force $\vec{F}$ (equation (11.6)) changes as the conductor is rotated from a position parallel to the field through an angle of 180° .
- 11.3 A long straight wire carries a current of 1.5 A. An electron travels with a velocity of $5 \times 10^4 \text{ m s}^{-1}$ parallel to the wire, 0.10 m from it, and in the same direction as the current. What force does the magnetic field of the current exert on the moving electron?
- 11.4 Figure 11.2 shows the current balance to be used in the experiments. The current element between the pole pieces is of length L and carries a current I . The magnetic induction between the pole pieces is $\vec{B}$. Use the principle of moments to obtain an expression among B , I , L , d , L_0 and mg at equilibrium.
- 11.5 What is the maximum torque on a coil $0.05 \text{ m} \times 0.12 \text{ m}$ which carries a current of 1 ampere in a field of magnetic induction $0.10 \text{ NA}^{-1} \text{ m}^{-1} (\text{Wb m}^{-2})$?
- 11.6 Why do you not have to worry about the weight of the wire when you calculate the force $\vec{F}$?

- 11.7 Could the force $\vec{F}$ be determined in this manner if an alternating current were used? Explain.
- 11.8 Explain in detail the reason for the downward force on the wire. Draw diagrams and use the cross product.
- 11.9 Find the current density required in a horizontal aluminium wire to make it "float" in the earth's magnetic field at the equator. The density of Al is $2.7 \times 10^3 \text{ kg m}^{-3}$. Assume that the earth's field is about $7 \times 10^{-5} \text{ Wb m}^{-2}$ and that the wire is orientated in the east-west direction.
- 11.10 A wire 0.60 m long, carrying 3.0 A is located in a region in which there is a magnetic field of 0.014 Wb m^{-2} . Find the magnitude of the magnetic force on this wire if the angle between the wire ($\vec{L}$) and the field $\vec{B}$ is, (a) 0° ; (b) 30° ; (c) 45° ; (d) 90° .
- 11.11 Suppose the magnetic fields of three magnets are in the opposite direction to those of the other three. Is any magnetic force exerted on the horizontal wire? Explain.
- 11.12 What is the effect of the earth's magnetic field on the results of the experiments?
- 11.13 Why is it not necessary to include the vertical pieces of wire in the magnetic field in the calculations?
- 11.14 The general equation for the force on a charge is $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. Interpret each symbol in this equation.

11.15 The following results were obtained for Experiment 6(a):

	I (A)	d (m)
$L_0 = 0.10 \text{ m}$ $L = 0.08 \text{ m}$ $m = 0.001 \text{ kg}$	0.1	0.0027
	0.4	0.0101
	0.8	0.0199
	1.2	0.0280
	1.5	0.0380
	1.8	0.0425
	2.0	0.0470

Analyse the data as described in § 11.4.

11.16 The following results were obtained for Experiment 6(b):

	L (m)	d (m)
$I = 1.2 \text{ A}$ $L_0 = 0.10 \text{ m}$ $m = 0.001 \text{ kg}$	0.02	0.0077
	0.04	0.0152
	0.06	0.0233
	0.08	0.0320

Analyse the data as suggested in § 11.5.

11.17 The following results were obtained for Experiment 6(c):

	B (Tesla)	d (m)
I = 1.2 A	0.005	0.0052
m = 0.001 kg	0.010	0.012
L = 0.08 m	0.015	0.0160
L ₀ = 0.10 m	0.020	0.0182
	0.025	0.0233
	0.030	0.0320

Analyse the data as suggested in § 11.6

$$F = \frac{mg}{L_0} = ILB$$

$$\frac{mg}{L_0} \propto L$$

$$(IL)L$$

$$F \propto I \quad L \text{ constant}$$

$$F \propto L \quad I \text{ constant}$$

$$F \propto B \quad I \text{ constant}$$

Experiments in Optics and Modern Physics

Chapter 12

Experiments in Geometrical Optics (exp. 7)

We all know what light is, but it is not easy to tell what it is.

Samuel Johnson (1709–1784)

12.1	Introduction	234
12.2	Sign conventions	234
12.3	The thin lens	235
12.4	Experiment 7(a) - focal length of a convex lens	239
12.5	Experiment 7(b) - smallest separation of object and real image	240
12.6	Experiment 7(c) - magnification of a convex lens	242
12.7	Experiment 7(d) - focal length of a concave lens (I)	243
12.8	Experiment 7(e) - focal length of a concave lens (II)	244
12.9	Refraction by a prism	245
12.10	The spectrometer	246
12.11	Experiment 7(f) - refraction by a prism	248
12.12	Questions	249

12.1 Introduction

Whenever a train of light waves travelling in one transparent medium strikes the surface of a second transparent medium of which the index of refraction differs from that of the first, two new wave trains are found to originate at the interface. The *reflected* wave travels back into the original medium while the *refracted* wave is propagated into the second medium. In this chapter our attention will be focused on the refracted wave with special reference to thin lenses.

12.2 Sign conventions

To use the equations derived below, a suitable sign convention must be used. The one summarised here applies equally well to reflecting and refracting surfaces.

1. All distances are measured along the axis *from* the reflecting or refracting surface *to* the point in question.
2. An object distance s is positive if the direction from the surface to the object is *opposite* to that of the *oncoming* light.
3. An image distance s' is positive if the direction from the surface to the image is the *same* as that of the *ongoing* light.
4. A radius of curvature R is positive if the direction from the surface to the centre of curvature is the *same* as that of the *ongoing* light.
5. An object or image dimension *above* the axis is positive while dimensions below the axis are negative.

12.3 The thin lens

In figure 12.1 P is an object at a distance s to the left of a spherical surface of radius R .

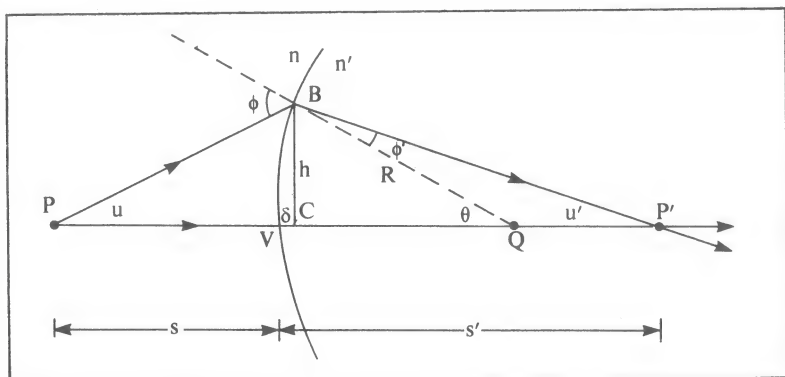


Figure 12.1 Object P in front of a spherical surface.

The indices n and n' are on the left and the right of the refracting surface respectively. Ray PV along the axis of the spherical surface is not deviated. Ray PB makes an angle u with the axis and is incident at an angle ϕ with the normal. It is refracted at an angle ϕ' . These two rays intersect at P' . The distance from the vertex V is s' .

From triangle PBQ

$$\phi = \theta + u. \quad (12.1)$$

If the angle u is small, u' , ϕ , ϕ' and θ are also small, and we may approximate both the sine and tangent of these angles by the angles.

Snell's law

$$n \sin \phi = n' \sin \phi' \quad (12.2)$$

therefore becomes

$$n \phi = n' \phi'. \quad (12.3)$$

From equation (12.1)

$$\begin{aligned} n u + n \theta &= n' \phi' \\ &= n' (\theta - u'), \end{aligned}$$

since $\theta = \phi' + u'$, from triangle $P'BQ$.

Hence

$$n u + n' u' = (n' - n) \theta. \quad (12.4)$$

From Figure 12.1 it also follows that

$$\begin{aligned} \tan u &= h/(s + \delta); & u &\simeq h/s \\ \tan u' &= h/(s' - \delta); & u' &\simeq h/s' \\ \tan \theta &= h/(R - \delta); & \theta &\simeq h/R \end{aligned} \quad (12.5)$$

since all angles are small and δ is negligible. Substituting into equation (12.4) we get

$$n/s + n'/s' = (n' - n)/R. \quad (12.6)$$

The problem of refraction at the surfaces of a lens is solved by considering the image of the first surface as the object of the second surface. PQ is the object in figure 12.2.

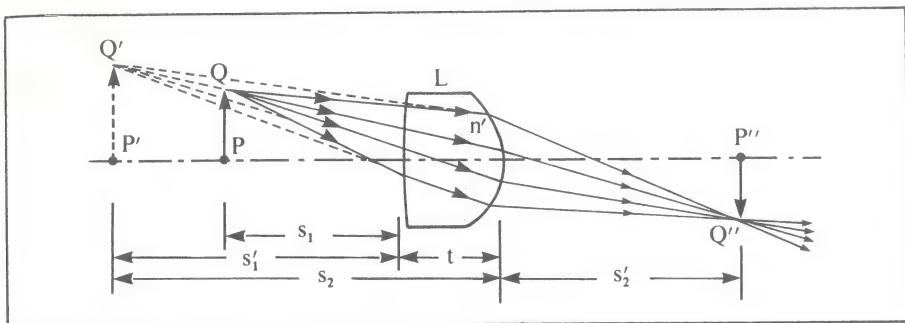


Figure 12.2 Refraction at the surfaces of a lens.

The first surface of lens L forms a virtual image of Q at Q' . This virtual image serves as a real object for the second surface which forms a real image of Q' at Q'' . For the first surface s_1 and s'_1 are the object and image distances respectively. The object distance for the second surface is s_2 while s'_2 is the image distance.

If the lens is thin, we assume that t is negligible in comparison with s_1 , s'_1 , s_2 and s'_2 . We therefore assume that s'_1 equals s_2 and measure object and image distances from either vertex of the lens. If the lens is surrounded by air ($n = 1.0$), equation (12.6) for the first surface yields

$$1/s_1 + n'/s'_1 = (n' - 1)/R_1. \quad (12.7)$$

Refraction at the second surface gives

$$n'/s_2 + 1/s'_2 = (1 - n')/R_2. \quad (12.8)$$

Since the lens is thin ($s_2 = -s'_1$), equations (12.7) and (12.8) give

$$1/s_1 + 1/s'_2 = (n' - 1)(1/R_1 - 1/R_2) \quad (12.9)$$

where s_1 is the object distance for the thin lens and s'_2 is the image distance. Equation (12.9) may be rewritten as

$$1/s + 1/s' = (n' - 1)(1/R_1 - 1/R_2). \quad (12.10)$$

If we set s equal to infinity, the focal length is found to be

$$1/f = (n' - 1)(1/R_1 - 1/R_2), \quad (12.11)$$

which is known as the *lens maker's equation*.

Therefore

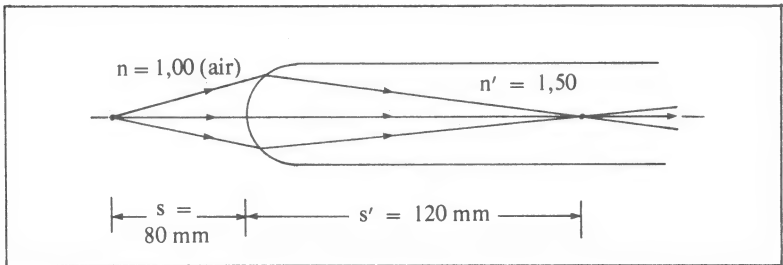
$$1/s + 1/s' = 1/f, \quad (12.12)$$

which is known as the *thin lens equation*.

The focal length of a *convex lens* is *positive* while that of a *concave lens* is *negative*.

Example 12.1

One end of a cylindrical glass rod is ground to a hemispherical surface of radius $R = 20$ mm as shown in the figure below.



Find the image distance of a point object on the axis of the rod, 80 mm to the left of the vertex. The rod is in air.

Solution:

$$n = 1, \quad n' = 1.5, \quad R = +20 \text{ mm}, \quad s = +80 \text{ mm}.$$

$$1/80 + 1.5/s' = (1.5 - 1)/20$$

$$\therefore \quad s' = +120 \text{ mm}.$$

The image is therefore formed at the *right* of the vertex (s' is positive) and at a distance 120 mm from it.

Example 12.2

Let the same rod be immersed in water of index 1.33, the other quantities having the same values as before. Find the image distance.

Solution:

$$1.33/80 + 1.5/s' = (1.5 - 1.33)/20$$

$$\therefore s' = -180 \text{ mm}$$

The fact that s' is negative means that the rays after refraction by the surface are not converging but appear to diverge from a point 180 mm to the *left* of the vertex. The surface forms a virtual image.

12.4 Experiment 7(a) – Focal length of a convex lens

An optical bench (figure 12.3) is used with an object A in the form of an illuminated mesh wire. The image is formed on a screen B by the lens C. (The concave lens D is removed from the bench). Focusing on the screen is facilitated by careful observation of the image. Try to find the sharpest image.

The experiment includes the following:

1. At each setting of a convenient series of values (at least seven) of the object distance s , measured with the aid of the optical bench scale E, the screen B is moved to obtain a corresponding sharply focused image. The values of s' and s are measured. Notice that if $s < f$, no image will be observed (virtual image).

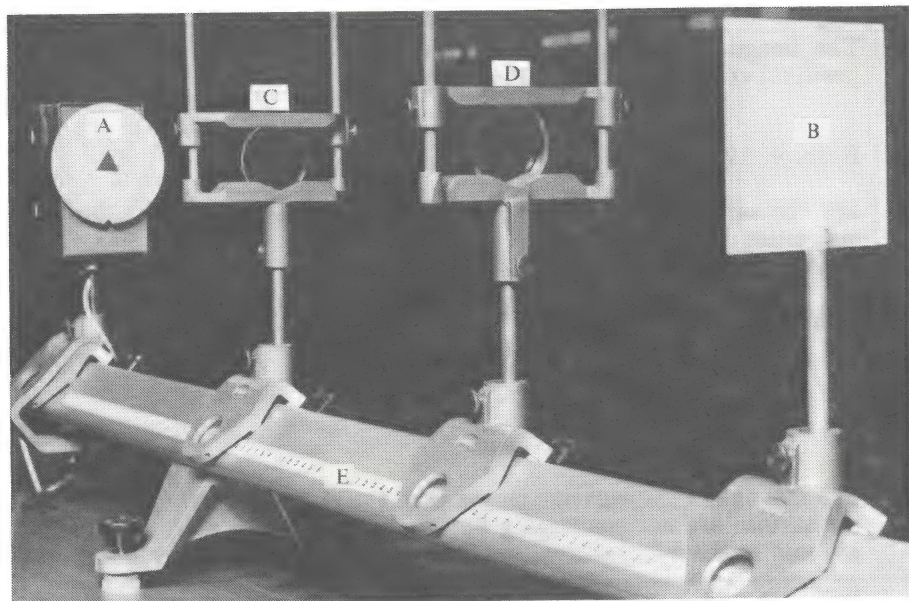


Figure 12.3 Optical bench.

2. Draw a suitable straight line graph, do a linear regression analysis and obtain f and s_f from the graph. Try to find a form where f is found from the slope of the graph rather than from the intercept.

12.5 Experiment 7(b) – smallest separation of object and real image

From the lens maker's equation it follows that

$$s + s' = ss'/f. \quad (12.13)$$

At a minimum or maximum

$$d(s+s')/ds = 0. \quad (12.14)$$

From equation (12.13)

$$\begin{aligned}\frac{d}{ds} (ss'/f) &= s'/f + s/f \frac{ds'}{ds} \\ &= 1/f(s' + s \frac{ds'}{ds}) = 0.\end{aligned}$$

$$\therefore ds'/ds = -s'/s. \quad (12.15)$$

From equation (12.14):

$$\begin{aligned}\frac{d}{ds} (s+s') &= 1 + \frac{ds'}{ds} \\ &= 0.\end{aligned} \quad (12.16)$$

Therefore at a maximum or a minimum

$$ds'/ds = -1. \quad (12.17)$$

Hence from equations (12.15) and (12.17)

$$s'/s = 1.$$

$$\therefore s' = s \quad (12.18)$$

Since either s or s' may be infinite, this condition corresponds to a *minimum* separation.

From equations (12.13) and (12.18) it follows that

$$\begin{aligned}s' &= 2f \\ &= s\end{aligned} \quad (12.19)$$

$$\therefore s' + s = 4f. \quad (12.20)$$

This shows that the object and the image have their smallest separation when the lens is midway between them with both s and s' equal to $2f$.

The experiment includes the following:

1. Determine s' as a function of s (equation 12.13).
2. Draw a graph of $(s+s')$ versus s and obtain f from the graph.

12.6 Experiment 7(c) – magnification of a convex lens

The magnification produced by a lens is the product of the magnifications of the two surfaces. The magnification due to the first refraction is (see Problem 12.2)

$$m_1 = -1 \times s'_1 / (ns_1) \quad (12.21)$$

and that due to the second refraction is

$$m_2 = -ns'_2 / (1 \times s_2). \quad (12.22)$$

The overall magnification of the lens is therefore

$$\begin{aligned} m &= [-s'_1 / (ns_1)] [-ns'_2 / s_2] \\ &= s'_1 s'_2 / (s_1 s_2). \end{aligned} \quad (12.23)$$

Since the lens is thin, $s_2 = -s'_1$ and

$$m = -s' / s. \quad (12.24)$$

All distances and surfaces are shown in figure 12.2.

The experiment includes the following:

1. Put the lens C at a distance s ($>f$) from the object A and obtain a well-focused image on the screen B (see figure 12.3).
2. Determine the distance between the highest and lowest horizontal wires on the screen. Count the number of horizontal wires.

3. Determine both s and s' .
4. Determine the distance between the same number of horizontal wires on the object.
5. Compare this value of m with the values obtained from the ratio of the heights of the image and the object.
6. Repeat the experiment for four other positions of the lens.

12.7 Experiment 7(d) – focal length of a concave lens (I)

The experiment includes the following:

1. Place the concave lens D of focal length f_d in contact with convex lens C of known focal length f_c (see figure 12.3).
2. At each setting of a convenient series of values (at least seven) of the object distance s_1 , measured with the aid of the optical bench scale E, the screen B is moved to obtain a corresponding sharply focused image. The values of s' are measured.
3. Draw a suitable straight line graph, do a linear regression analysis and obtain f , the focal length of the combination and s_f from the graph.
4. Calculate f_d and s_d from the formula.

$$1/f = 1/f_d + 1/f_c. \qquad \text{(Problem 12.7)}$$

12.8 Experiment 7(e) – focal length of a concave lens (II)

In this experiment the concave and a convex lens are used, but they are not in contact. The image I_1 formed by the convex lens L_c serves as an object for the concave lens L_d which forms a final image I_2 (see figure 12.4).

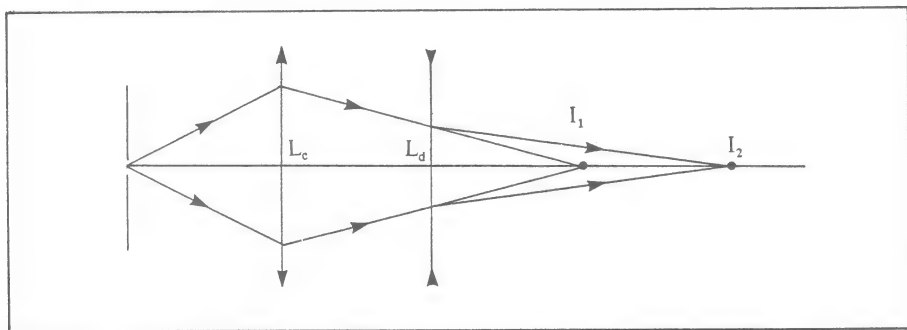


Figure 12.4 Convex and concave lens system.

The experiment includes the following:

1. Put the convex lens on the bench and find the image at I_1 . Record the position of I_1 .
2. Interpose the concave lens L_d between L_c and I_1 .
3. Find the position of the final image I_2 .
4. Determine s and s' of the concave lens. Apply the sign conventions to find the proper signs.
5. Repeat steps (1), (2), (3) and (4) at least five times with the convex lens in different positions.

6. Draw a suitable straight line graph, do a linear regression analysis and obtain f_d and s_d from it.

12.9 Refraction by a prism

Consider a light ray incident at an angle ϕ_1 on one face of a prism as shown in figure 12.5.

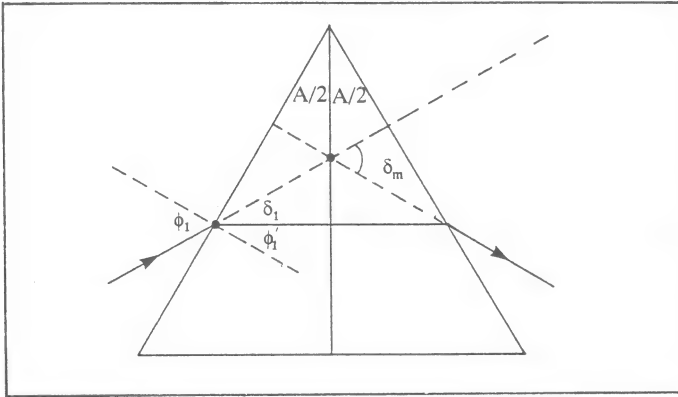


Figure 12.5 Prism

The index of refraction of the prism is n and the top angle A . The problem is to find the *angle of deviation* δ . In principle it is possible to solve the problem by applying Snell's law at the two surfaces. The expression for δ thus obtained is very complicated. As the angle of incidence is decreased from a large value, the angle of deviation decreases at first and then increases. δ is a minimum when the ray passes symmetrically through the prism as shown in figure 12.5. In this case the angle δ_m is called the *angle of minimum deviation*. From the geometry of the figure it follows that

$$\phi_1' = A/2 \quad \text{and} \quad \delta_1 = \delta_m/2.$$

Therefore

$$\begin{aligned}
 \phi_1 &= \phi_1' + \delta_1 \\
 &= A/2 + \delta_m/2 \\
 &= (A + \delta_m)/2.
 \end{aligned}
 \tag{12.25}$$

From Snell's law

$$\sin \phi_1 = n \sin \phi_1' \tag{12.26}$$

(we assume the prism is in air) and equation (12.25) it follows that

$$\sin[(A + \delta_m)/2] = n \sin(A/2)$$

Therefore

$$n = \sin[(A + \delta_m)/2] / \sin(A/2). \tag{12.27}$$

12.10 The spectrometer

The spectrometer is an apparatus for the production of spectra and consists essentially of four parts (see figure 12.6).

1. The collimator C with adjustable slit. It provides a parallel beam of light from the source N.
2. The rotatable prism table P.
3. The telescope T which receives the dispersed rays. The eyepiece is provided with cross wire.
4. The scale S. The angle between different positions of T or the angle between C and T can be determined with the aid of a vernier scale (see § 5.3).

The slit is adjustable in width. A wide slit admits more light but reduces the precision of the readings. The collimator will usually be found adjusted.

If not, the draw tube should be adjusted until the image of the slit is formed. Focus the eyepiece of the telescope on the cross wires until they are clearly visible. Then adjust the telescope alone until an image of a distant object falls in the plane of the cross wires without parallax. The telescope is then adjusted to accept parallel rays. Bring the telescope into line with the collimator and adjust the collimator until an image of the slit appears in the plane of the telescope cross wires. The light rays between collimator and telescope are now parallel. This is an important condition for proper use of the instrument. Focus the telescope on the slit by means of the focusing screw.

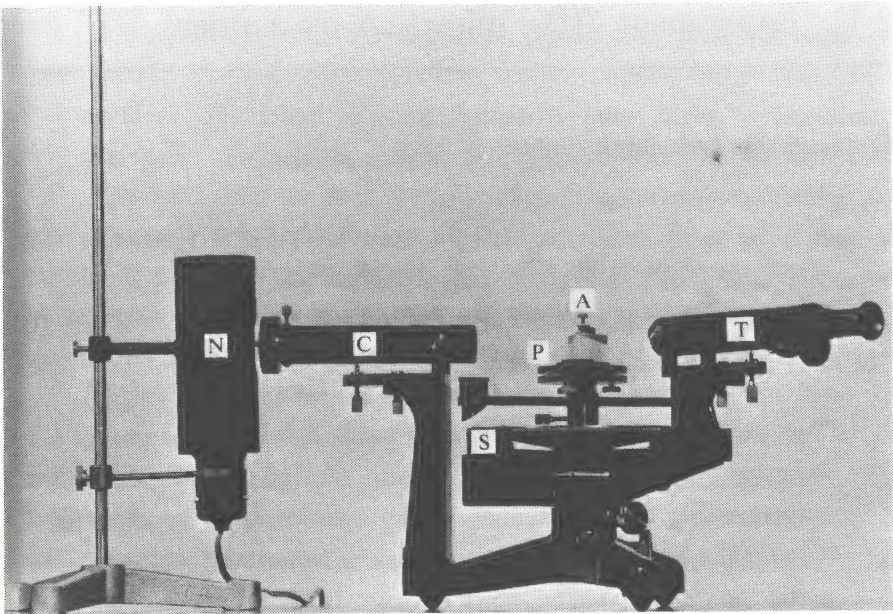


Figure 12.6 Spectrometer.

Most light sources are *polychromatic* which means that they are a mixture of different wavelengths or colours. Ordinary light bulbs are the most

commonly available sources. The heated filament produces a *continuous spectrum*. The spectrum can easily be investigated with a prism. A *line spectrum* is produced by a gas discharge tube. The characteristics of the spectrum depend on the gas in the tube. A high voltage (~ 5000 V) must be applied across the tube. A fluorescent light bulb produces both a continuous spectrum as well as a line spectrum. A *monochromatic* light source emits a beam of one particular wavelength or colour. A sodium vapour lamp gives off yellow light that is almost monochromatic within the visible region. The two sodium lines have wavelengths of 5889.95 and 5895.92 Å.

12.11 Experiment 7(f) – refraction by a prism

The experiment includes the following:

1. Direct the top angle of the prism A towards the collimator C and clamp the prism table. The apex of the prism must be at the centre of the prism table. Obtain the positions of the reflected images on both sides of the prism. Repeat five times and calculate the average value of the top angle. A sodium lamp is used as a light source.
2. Two angles are read on the circular scale, one corresponding to the direction of the undeviated beam of light and the other corresponding to the direction of the light deviated by the prism. The first angle is read off the scale before mounting the prism. The prism is then mounted as shown in figure 12.7.

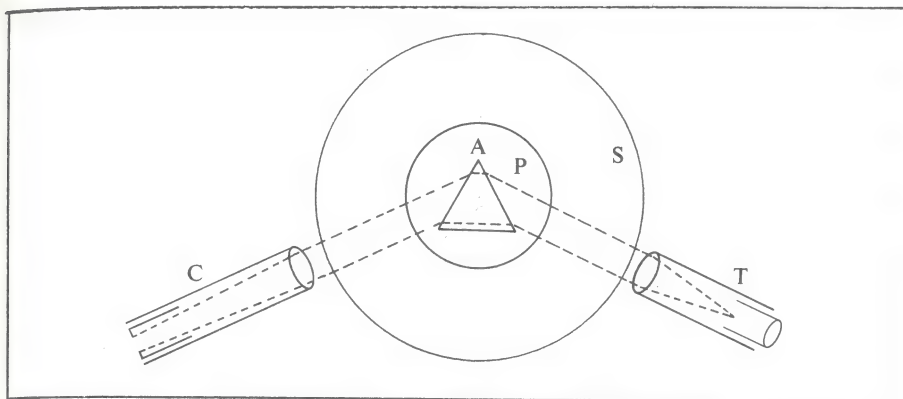


Figure 12.7 Prism mounted on the spectrometer.

3. The telescope is rotated in the proper direction until the slit image is found. The angle through which the telescope is turned gives the angle of deviation, but not necessarily the angle of minimum deviation. To find this angle the prism table is rotated until the image of the slit comes to a standstill and then reverses its motion as rotation of the prism table is continued. The angle of minimum deviation is the difference between the initial reading and the angle at which the image comes to a standstill. Repeat the procedure at least five times and calculate the index of refraction of the glass for the light used.

12.12 Questions

- 12.1 Explain what is meant by real and virtual images; by real and virtual objects. How can a virtual object be obtained? Illustrate with appropriate diagrams.

- 12.2 Draw a diagram similar to figure 12.1 and show that the magnification is given by $m = -ns'/sn'$.
- 12.3 Show how you would determine graphically the position and size of the image of an object formed by a thin concave lens.
- 12.4 Draw a complete ray diagram for the determination of the focal length of a concave lens.
- 12.5 State the basic law for refraction at a spherical surface.
- 12.6 Consider the equation

$$1/s + 1/s' = 1/f.$$

Measurements are made of s and s' . What quantities should be plotted to obtain a straight line from which f can be obtained?

- 12.7 a) Prove that when two thin lenses of focal lengths f_1 and f_2 are placed in contact, the focal length f of the combination is given by the relation

$$1/f = 1/f_1 + 1/f_2.$$

- b) A converging meniscus lens has an index of refraction of 1.50 and the radii of its surfaces are 5 and 10 cm. The concave surface is placed upward and filled with water. What is the focal length of the water-glass combination?

- 12.8 A lens casts an image of an object on a screen placed 12 cm from the lens. When the lens is moved 2 cm further from the object, the screen must be moved 2 cm closer to the object in order to bring it into focus. What is the focal length of the lens?
- 12.9 An equiconvex thin lens made of glass of index 1.50 has a focal length in air of 30 cm. The lens is sealed into an opening in one end of a tank filled with water ($n = 1.33$). At the end of the tank opposite the lens is a plane mirror, 80 cm distant from the lens. Find the position of the image formed by the lens–water–mirror system, of a small object outside the tank on the lens axis and 90 cm from the lens. Is the image real or virtual? Erect or inverted?
- 12.10 Three thin lenses, each of focal length 20 cm, are aligned on a common axis and are separated by 30 cm. Find the position of the image of a small object on the axis, 60 cm to the left of the first lens.
- 12.11 A converging lens has a focal length of 10 cm. For object distances of 30 cm, 20 cm, 15 cm and 5 cm determine the
- a) image position,
 - b) magnification,
 - c) whether the image is real or virtual,
 - d) whether the image is erect or inverted.
- 12.12 A convergent lens has a focal length of 0.60 m. Find the position of the object to produce an image
- a) real and three times larger;
 - b) real and one-third as large;
 - c) virtual and three times larger.

- 12.13 Construct a diagram similar to figure 12.1 and derive equation (12.6) if R is negative and $n > n'$.
- 12.14 Repeat 12.13 but with $R < 0$ and $n < n'$.
- 12.15 Repeat 12.13 but with $R > 0$ and $n > n'$.
- 12.16 What will be seen through the telescope if ordinary white light is used? Explain.
- 12.17 Show that the top angle of the prism is half the angle between the two reflected beams.
- 12.18 What is the purpose of the collimator on the spectrometer?
- 12.19 A light ray strikes a parallel-sided glass plate of thickness t at an angle ϕ . Show that the lateral displacement D of the emergent beam is given by the relation

$$D = t \sin(\phi - \phi') / \cos \phi'$$

where ϕ' is the angle of refraction of the ray within the glass.

- 12.20 Which colour is deviated most? Which colour is deviated least? For which colour has glass its largest index of refraction?
- 12.21 How do you expect a plot of angle of deviation as a function of angle of incidence to look?
- 12.22 What is the angle of minimum deviation of an equiangular prism having a refractive index of 1.414?

12.23 The following results were obtained for Experiment 7(a).

Position of object (m)	Position of lens (m)	Position of screen (m)
0.10	0.602	0.727
0.10	0.503	0.630
0.10	0.458	0.592
0.10	0.351	0.522
0.10	0.300	0.495
0.10	0.252	0.560
0.10	0.236	0.674

Analyse the results as indicated in § 12.5.

12.24 The following results were obtained for Experiment 7(c):

Position of object (m)	Position of lens (m)	Position of screen (m)	Distance between 1st and 6th wire on screen (m)
0.10	0.602	0.727	0.0028
0.10	0.503	0.630	0.0034
0.10	0.458	0.592	0.0046
0.10	0.351	0.522	0.0077
0.10	0.306	0.495	0.0114
0.10	0.252	0.560	0.0223
0.10	0.236	0.674	0.0382

Distance between 1st and 6th wire on object = 0.0113 m.

Analyse the results as suggested in § 12.7

12.25 The following results were obtained for Experiment 7(d):

Position of object (m)	Position of lens (m)	Position of image (m)
0.100	0.500	1.688
0.100	0.551	1.450
0.100	0.602	1.347
0.100	0.653	1.324
0.100	0.701	1.282
0.100	0.749	1.309
0.100	0.849	1.336
0.100	0.952	1.411
0.100	1.090	1.529
0.100	1.301	1.721
0.100	1.605	1.998

The focal length of the convex lens is (0.10 ± 0.01) m.

Analyse the results as prescribed in § 12.7.

12.26 The following results were obtained for Experiment 7(e):

L_c (m)	L_d (m)	I_1 (m)	I_2 (m)
0.50	0.580	0.627	0.650
0.40	0.500	0.530	0.538
0.35	0.400	0.492	0.640
0.25	0.280	0.422	2.90
0.20	0.281	0.395	0.753
0.15	0.382	0.460	0.550
0.13	0.501	0.574	0.650

The object is always at the zero position.

Analyse the results as suggested in § 12.8.

Chapter 13

Experiments in physical optics (exp. 8)

Where there is a great deal of light, the shadows are deeper.

Johann Wolfgang von Goethe (1749–1832)

13.1	Introduction	256
13.2	Diffraction by a single slit	256
13.3	Diffraction by a circular aperture	258
13.4	Diffraction by two identical parallel slits	259
13.5	Diffraction gratings	261
13.6	Electron diffraction	262
13.7	Experiment 8(a) - single slit diffraction	266
13.8	Experiment 8(b) - diffraction pattern of blood cells	268
13.9	Experiment 8(c) - diffraction by a grating	268
13.10	Experiment 8(d) - electron diffraction	269
13.11	Questions	271

13.1 Introduction

In this chapter the phenomena of interference and diffraction are discussed. The principles of geometrical optics, discussed in Chapter 12, cannot explain these phenomena. Instead, the more fundamental point of view that light is a wave motion has to be adopted. The effect of a number of wave trains arriving at one point on a screen depends upon the phases of the waves as well as their amplitudes. This part of physics is called *physical optics*.

13.2 Diffraction by a single slit

Consider a long and very narrow slit with the incident waves normal to the plane of the slit (figure 13.1). The width b of the slit AE should be comparable to the wavelength of the incident waves.

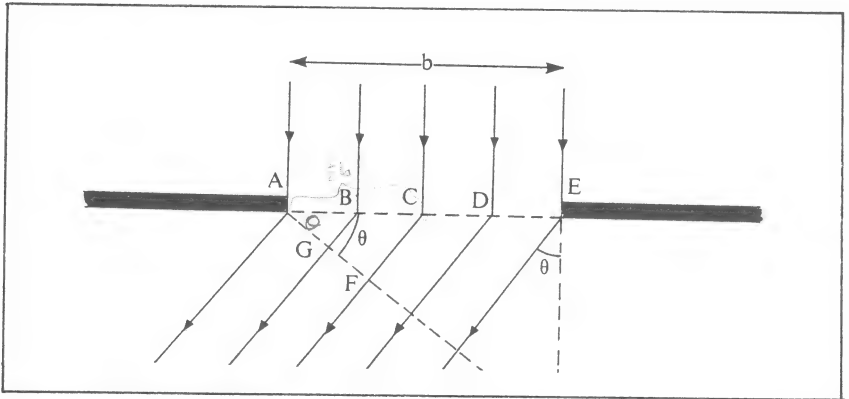


Figure 13.1 Single narrow slit of width b .

According to Huygens' principle all points of the incident wave falling on the slit act as secondary sources of waves. These waves are called *diffracted waves* in the present case. If these waves are diffracted at an angle θ (figure 13.1) it follows that the path difference of ray A coming from the one edge and ray C coming from the centre of the slit is

$$CF = \frac{1}{2}b\sin\theta. \quad (13.1)$$

If this path difference is equal to $(n + \frac{1}{2})\lambda$ where λ is the wavelength of the light (monochromatic, i.e. only one wavelength) and $n = 0, 1, 2, 3, \dots$, the interference between the two rays is totally destructive. Therefore if

$$\frac{1}{2}b\sin\theta = (n + \frac{1}{2})\lambda$$

or

$$b\sin\theta = (2n + 1)\lambda$$

$$b\sin\theta = N\lambda \quad ; \quad N = 1, 3, 5, 7, \dots \quad (13.2)$$

these two rays as well as all other pairs originating from points separated by $\frac{1}{2}b$ interfere destructively. Let us consider points A and B separated by $b/4$. The path difference is now

$$BG = \frac{1}{4}b\sin\theta$$

and for destructive interference

$$\frac{1}{4}b\sin\theta = (n + \frac{1}{2})\lambda$$

or

$$b\sin\theta = N\lambda \quad ; \quad N = 2, 6, 10, \dots \quad (13.3)$$

By extending this procedure all integers can be included. It is therefore concluded that *minima* in the intensity occur when

$$b\sin\theta = N\lambda \quad ; \quad N = \pm 1, \pm 2, \pm 3, \dots \quad (13.4)$$

The *maxima* occur approximately between minima, i.e. at $N = \pm 3/2, \pm 5/2, \dots$ The intensity distribution for a single narrow slit is shown in figure 13.2.

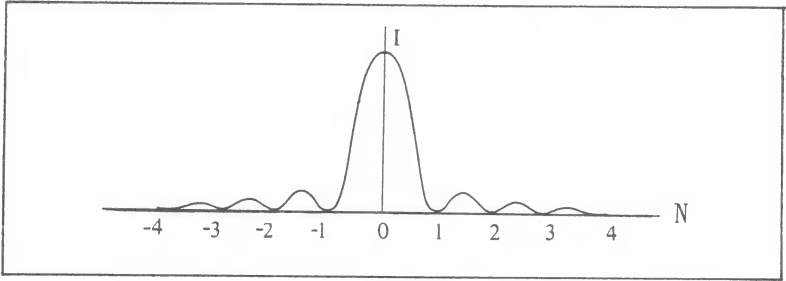


Figure 13.2 Diffraction intensity distribution for a single narrow slit.

The other maxima are considerably smaller than the centre ($N = 0$) maximum and the intensity drops fast with increasing $|N|$. This happens because wavelets from some parts of the slit cancel even at the maxima. This partial cancellation becomes more and more complete as the angle θ increases.

13.3 Diffraction by a circular aperture

If we use a small circular opening of diameter d instead of the single slit, a circular diffraction pattern will be observed instead of the linear diffraction pattern. The equation for the *minima* has the same form as that for the linear case, viz.

$$d \sin \theta = N \lambda \quad (13.5)$$

but the circular geometry results in

$$N = 1.220; 2.233; 3.238; 4.241; \dots$$

13.4 Diffraction by two identical parallel slits

Consider two slits of width b separated a distance a as shown in figure 13.3. If the slits are illuminated by parallel rays of monochromatic light, the two slits serve as coherent sources, i.e. at the slits the incident light waves are always in phase. In the θ direction two sets of diffracted waves, one from each slit, are observed and interference of these waves will take place. The observed pattern is therefore a superposition of the diffraction pattern from a single slit and the interference pattern resulting from both slits. According to the theory of *interference*, maxima will occur at directions given by

$$a \sin \theta = n \lambda \quad ; \quad n = 0, \pm 1, \pm 2, \pm 3, \dots \quad (13.6)$$

The zero's of the *diffraction* pattern are given by

$$b \sin \theta = N \lambda \quad ; \quad N = \pm 1, \pm 2, \pm 3, \dots \quad (13.7)$$

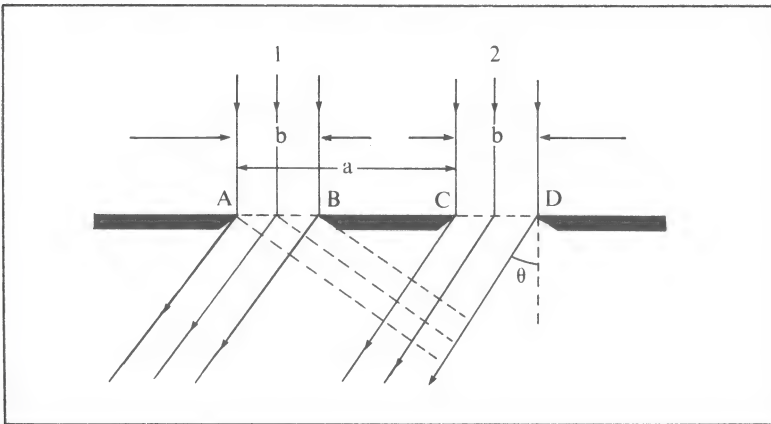


Figure 13.3 Two narrow slits of width b and a distance a apart.

Since a is larger than b , the zero's of the diffraction pattern are more widely spaced than the maxima of the interference pattern. *The single slit diffraction pattern forms an envelope for the interference pattern* or the interference pattern is modulated by the diffraction pattern. Therefore the intensity of the interference peaks becomes smaller further away from the centre of the pattern as illustrated in figure 13.4.

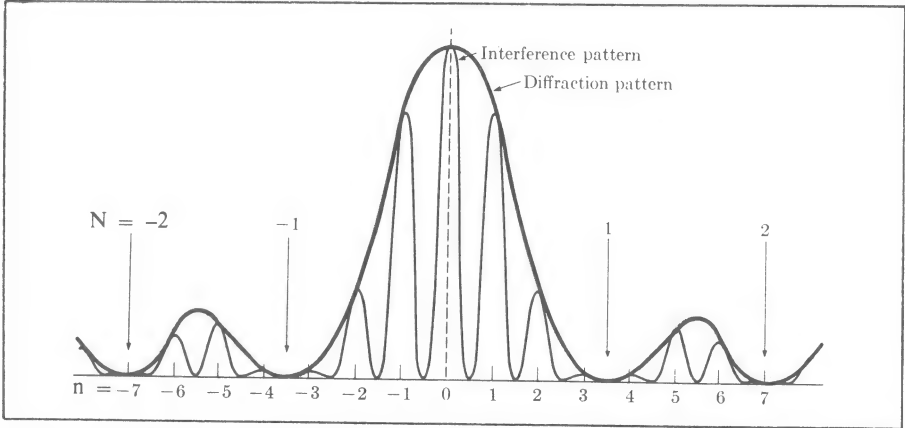


Figure 13.4 Interference and diffraction patterns of a double slit.

The angular width of the diffraction pattern depends on the size of the slit relative to the wavelength of the light. If the width of the slit $b = \lambda$, it follows that for the first maximum

$$\theta = \sin^{-1}(1) = 90^\circ$$

The central peak of the diffraction envelope is therefore very broad. On the other hand if $b = 20\lambda$,

$$\theta = \sin^{-1}(1/20) = 2.9^\circ$$

and the peaks are relatively narrow. It is difficult to interpret such a two slit spectrum since the interference lines are not observable in the regions where the diffraction minima occur.

13.5 Diffraction gratings

A grating consists of a large number N of identical slits of width b , equally spaced. The distance between two adjacent slits is a as shown in figure 13.5.

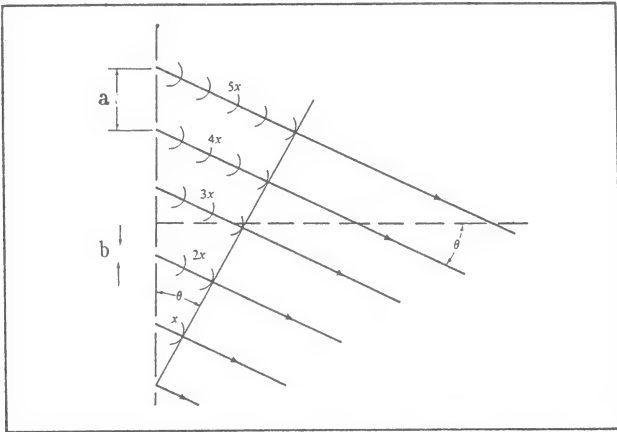


Figure 13.5 Diffraction grating.

In the direction θ the interference pattern of N sources (one per slit) modulated by the diffraction pattern of one slit will be observed. If the angle θ is such that

$$a \sin \theta = n\lambda, \quad n = 0, \pm 1, \pm 2 \dots \quad (13.8)$$

a *maximum* will be observed. The intensity of this maximum will be much higher than the intensity of the corresponding maximum in the two slit experiment since the amplitudes of all N slits add in the case of a grating.

Similarly to two slit interference, if $b \simeq \lambda$ the diffraction envelope is so broad that no variation in intensity between successive interference peaks is observed. If $b \gg \lambda$, the diffraction envelope becomes important and a pattern similar to figure 13.4 will be observed, i.e. the interference peaks differ in intensity.

13.6 Electron diffraction

De Broglie postulated that if the momentum of a particle is p , the wavelength associated with the particle is given by

$$\lambda = h/p \quad (13.9)$$

where h is Planck's constant (6.63×10^{-34} J s). If an electron has been accelerated from rest through a potential difference V , its kinetic energy is given by

$$\frac{1}{2}mv^2 = eV. \quad (13.10)$$

The momentum of the electron is

$$p = (2meV)^{\frac{1}{2}} \quad (13.11)$$

and the wavelength of the electron is

$$\lambda = h/(2meV)^{\frac{1}{2}}. \quad (13.12)$$

If for example the accelerating potential is 20 volt, the wavelength of the electron is

$$\lambda = 2.7 \times 10^{-10} \text{ m.}$$

Normally the grating spacings of commercially available gratings are of the order of $2.5 \times 10^{-6} \text{ m}$. The wavelengths of accelerated electrons are therefore much smaller than ordinary grating spacings and a grating with spacing much smaller is required to observe diffraction of matter waves. In 1912 Max von Laue suggested that crystals with their regularly spaced planes of atoms might serve as gratings for waves of wavelengths comparable with the spacings between the planes of the crystal lattice (usually of the order 10^{-10} m). Davison and Germer observed the first diffraction of electrons in 1927. The interference is more complicated because the sources of radiation are not simple parallel lines or slits, but individual atoms distributed in three dimensions, as illustrated in figure 13.6.

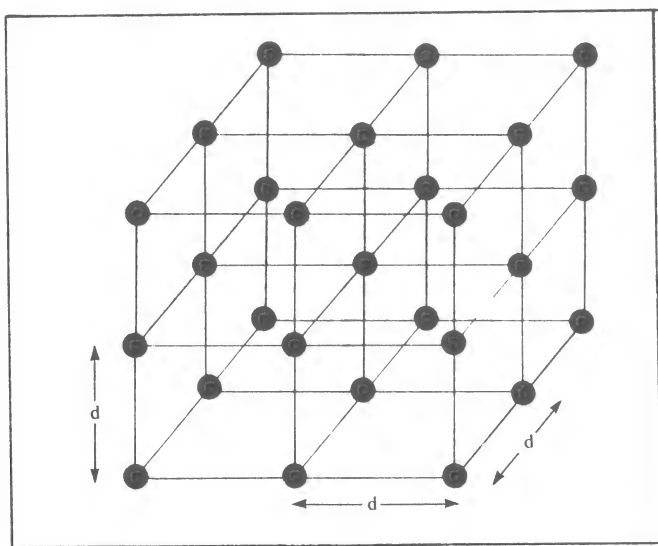


Figure 13.6 Atoms distributed in three dimensions.

The basic principle is however the same. Each point source emits a wavelet which may interfere with wavelets from other sources. The diffraction pattern for a single crystal consists of spots. The reason is that, among other things, the cross-section of the incident beam is small and thus also a spot.

Consider an electron beam falling on a crystal making an angle θ with a certain family of planes as illustrated in figure 13.7.

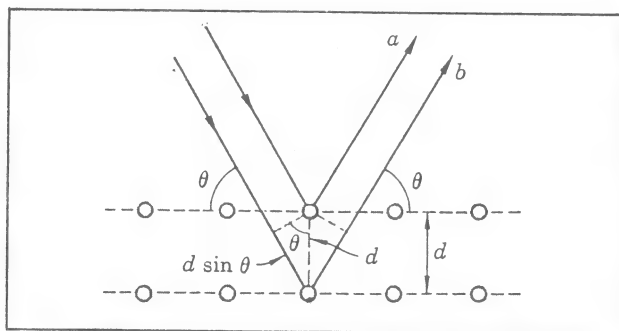


Figure 13.7 Electron beam incident on two planes of atoms, a distance d apart

We investigate the scattered electrons along a symmetric direction which also makes an angle θ with the family of planes. If d is the separation of the consecutive planes, maximum intensity of the scattered waves is observed if

$$2d\sin\theta = n\lambda \quad ; \quad n = 1, 2, 3, \dots \quad (13.13)$$

where n is an integer (the Bragg condition). For fixed planes (or fixed d) and wavelength λ , changing θ alternately produces positions of maximum and minimum intensity, corresponding to constructive or destructive interference. For a given direction of incidence relative to the whole crystal, we have a series of maxima corresponding to scattering from all

families of planes for which equation (13.13) holds. The maxima are in different directions because of the different orientations of the families of planes. If a screen is interposed in the path of the scattered rays from a single crystal as shown in figure 13.8, a regular pattern characteristic of the crystal, appears.

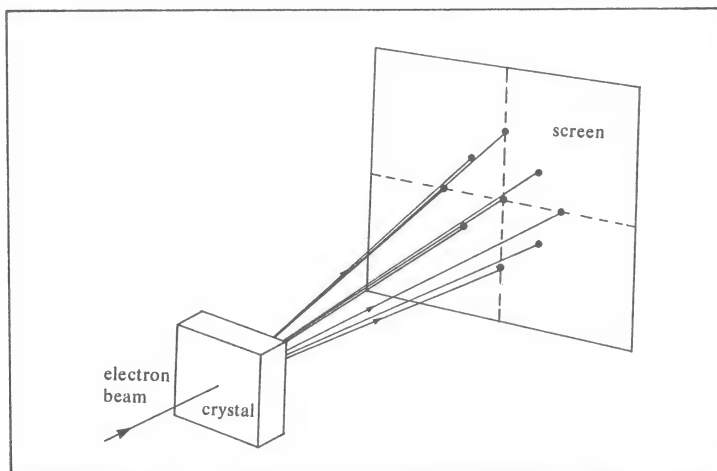


Figure 13.8 Scattering rays from a crystal.

Each dot in the pattern corresponds to the direction of scattering from a family of planes.

If the material causing the scattering is a powder consisting of a large number of small crystals randomly orientated, the same characteristic pattern will be generated by each small crystal. The randomness of the orientations of the crystals results in the different orientations of the patterns about the direction of the incident beam. Since the number of small crystals is large, the observed pattern consists of concentric circles (see figure 13.9). Each circle corresponds to a spot on the single crystal picture.

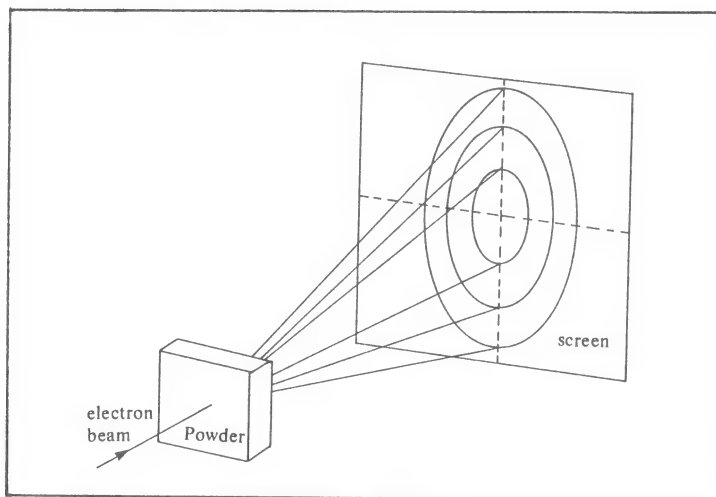


Figure 13.9 Scattering rays from a powdered sample.

13.7 Experiment 8(a) – single slit diffraction

We use a helium–neon laser (see figure 13.10) as a source of monochromatic light ($\lambda = 6328 \text{ \AA}$). Instead of a single slit a human hair is used. The diffraction pattern which is produced is the same as that which would be produced if a single slit, the width of the hair, were used.

The experiment includes the following:

1. Insert the supplied slide with the human hair in the holder at the front of the laser tube. If the hair is in the laser beam, the diffraction pattern will be observed on the screen.
2. Measure the distance between the first, second and third minima on either side of the central maximum.
3. Measure the distance from the hair to the wall.

4. Calculate an average value for the thickness of the hair.
5. No graphs or error analyses are required.

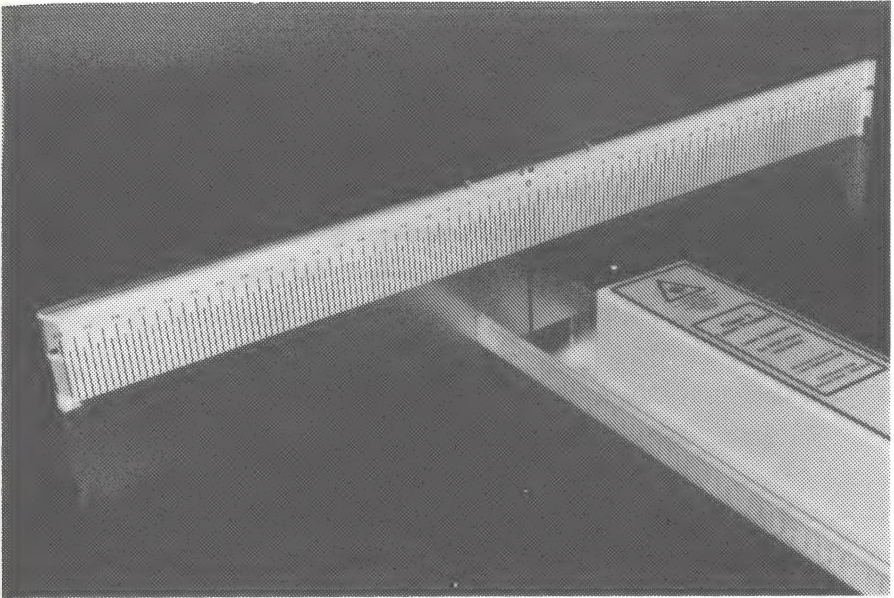


Figure 13.10 Helium–neon laser and screen.

13.8 Experiment 8(b) – diffraction pattern of blood cells

The spherical blood cells serve as obstacles causing diffraction of the laser beam. The equation for the minima of the pattern consisting of concentric rings, is exactly the same as that for a single circular aperture (§ 13.3).

The experiment includes the following:

1. Insert the glass slide, covered by a thin layer of blood in the holder at the front of the laser tube. Switch the laser beam on and move the slide until a pattern characterised by two or three dark rings is observed on the screen, about 10 cm from the tube.
2. Measure the diameters of the rings and the distance from the screen to the glass slide.
3. Calculate the diameter of a red blood cell.
4. No graphs or error analyses are required.

13.9 Experiment 8(c) – diffraction by a grating

The experiment includes the following:

1. Insert the grating which produces the largest number of diffraction spots in the holder at the front of the laser beam. The diffraction pattern can be observed on the wall about 50 cm from the grating.

2. Determine the centre of the pattern O and measure the positions of the maxima from O.
3. Measure the distance between the screen and the grating.
4. Draw a suitable straight line graph to determine the wavelength of the laser beam. The number of slits per unit length of the grating is given on the grating.
5. Do a linear regression analysis of the data.

13.10 Experiment 8(d) – electron diffraction

The apparatus is shown in figure 13.11. Electrons, accelerated between the cathode C and anode A inside an evacuated glass tube G, are diffracted by a thin piece of polycrystalline material B. The diffraction pattern is observed as concentric circles on screen S.

The experiment includes the following:

1. Adjust the potential difference across the cathode and the anode until the concentric circles are clearly visible on the screen.
2. Calculate the wavelength of the electrons.
3. Measure the diameter of the first circle and the distance between S and B. Repeat this for 2 other settings of the potential difference.
4. Calculate the average distance between planes in the polycrystalline material.

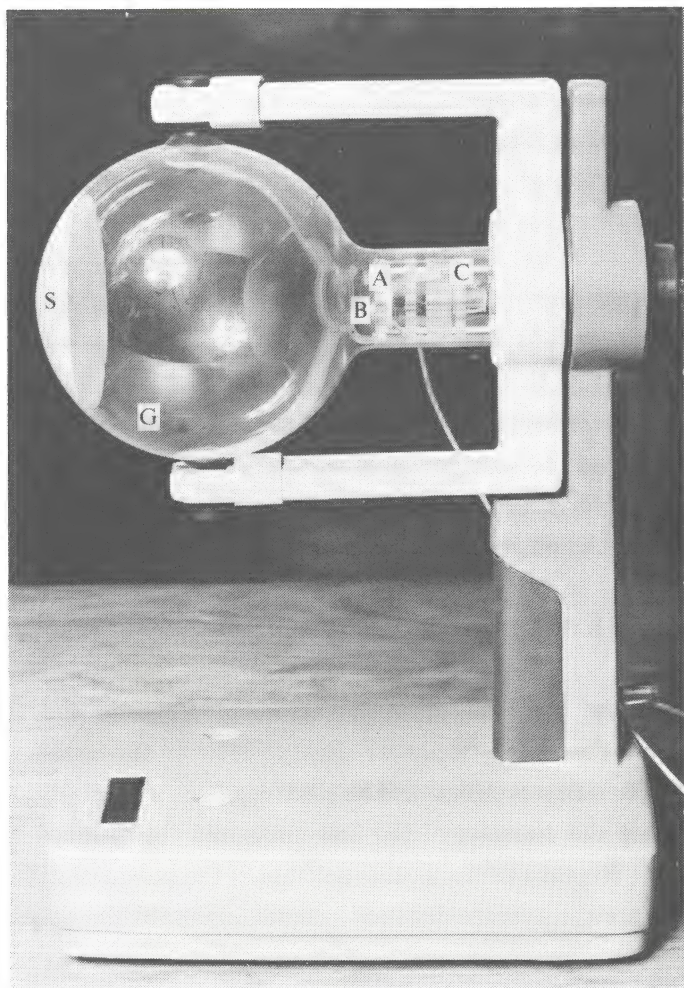


Figure 13.11 Electron diffraction apparatus.

13.11 Questions

- 13.1 What is the effect on the diffraction pattern from a double slit if
- a) the distance between slits is kept unchanged and the slit width is varied?
 - b) the slit width is kept fixed and the distance between slits is varied?
- 13.2 What is meant by coherent light? What is meant by monochromatic light? Give examples of each.
- 13.3 If a separate light source is used for each slit of a double slit diffraction experiment, would a diffraction pattern be observed? Explain.
- 13.4 State clearly de Broglie's hypothesis regarding the wave nature of matter.
- 13.5 What is the relation connecting the de Broglie wavelength (λ), the momentum of a particle (p) and Planck's constant (h)?
- 13.6 Explain the principle behind the Davison-Germer experiment by drawing comparisons with the diffraction of electromagnetic waves.
- 13.7 Why does the electron diffraction pattern you have obtained consist of rings? How do the radii of these rings vary with the wavelength of the electrons?

- 13.8 Derive the relationship between the de Broglie wavelength of the electrons and the accelerating voltage.
- 13.9 In the electron diffraction experiment a graph of λ^2 against $1/V$ is plotted where λ is in Å and V in volts. What is the significance of the slope?
- 13.10 Mention some instances in which diffraction of sound waves is observed.
- 13.11 Describe the diffraction pattern observed when a slit is illuminated by white light.
- 13.12 Is the double slit experiment a pure interference effect or a mixture of interference and diffraction? Explain.
- 13.13 How should the ratio a/b be chosen in a double slit experiment so that diffraction effects are negligible?
- 13.14 In a double slit diffraction pattern the third principle maximum is missing because it coincides with the first diffraction zero. Find the ratio a/b .
- 13.15 An observer looks at a source through a grating having 364 slits per cm. If λ is 550 nm how many images of the source can be seen?

13.16 The following data were obtained for experiment 8(c):

Distance from grating to the screen $D = 40$ cm.

Number of slits per unit length of the grid $N = 2.953 \times 10^5 \text{ m}^{-1}$.

n	Position of maximum d (cm)
-3	-28.0
-2	-17.0
-1	-8.2
0	0.0
1	8.4
2	18.3
3	33.5

Analyse the results as indicated in § 13.9.

13.17 Repeat question 13.16 with the following data:

$D = 80$ cm

$N = 9.449 \times 10^4 \text{ slits m}^{-1}$.

n	Position of maximum d (cm)
-4	-20.5
-3	-15.2
-2	-10.1
-1	-5.1
0	0.0
1	5.0
2	10.2
3	15.5
4	21.0

13.18 Repeat question 13.16 with the following data:

$$D = 60 \text{ cm}$$

$$N = 2.953 \times 10^5 \text{ slits m}^{-1}$$

n	Position of maximum d (cm)
-3	-43.1
-2	-25.5
-1	-12.0
0	0.0
1	12.3
2	26.2
3	45.5

Chapter 14

Photoelectric effect (exp. 9)

What is now proved, was once only imagined.

William Blake (1757–1827)

14.1	Introduction	276
14.2	Theory	276
14.3	Experiment 9 - photoelectric effect	278
14.4	Questions	282

14.1 Introduction

The phenomena studied in Chapter 13 can be explained satisfactorily by using the wave picture of light. We know, however, that light exhibits both wave like and particle-like characteristics and that the particular aspect exhibited in a given case depends on the details of the prevailing physical situations. Certain experiments will bring out the particle-like features of light and others its wave like features. In this chapter we shall be concerned with the photoelectric effect which is a clear demonstration of the particle nature of light.

14.2 Theory

A clean metal surface, illuminated by a light beam of *sufficiently high frequency* f *immediately* ejects electrons with a variety of kinetic energies which range up to a maximum value E . The corresponding *minimum* frequency is called the *threshold frequency* of the particular surface. This phenomenon can be studied in detail with the apparatus shown in figure 14.1. S is the source of radiation of variable and known frequency f and intensity I . T is an emitting electrode of the material being studied and C is a collecting electrode. Both electrodes are contained in an evacuated glass tube P with a quartz window R that permits the passage of ultraviolet and visible light. The electric circuit allows the electrodes to be maintained at different known potentials and permits the measurement of any current between the electrodes.

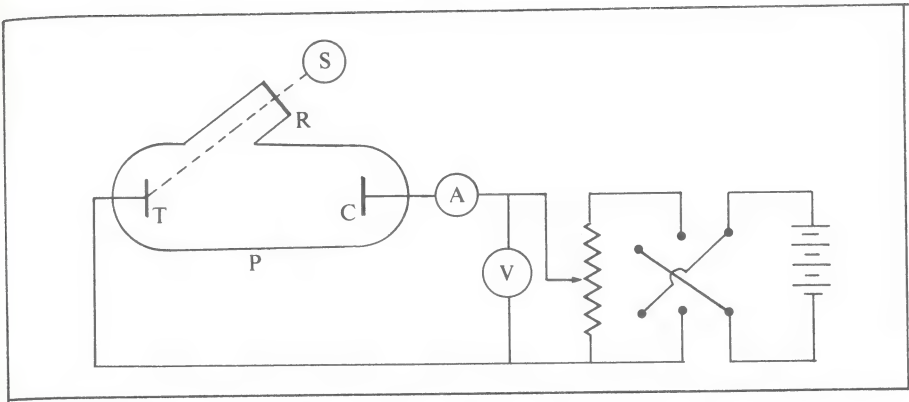


Figure 14.1 Schematic diagram of the apparatus for the photoelectric effect.

Increasing the intensity of the light beam does not affect the kinetic energy of the ejected electrons, but the number of electrons ejected per unit time is proportional to the intensity. The maximum kinetic energy of these electrons depends on the frequency f of the light according to what is called the Einstein photoelectric equation

$$E = hf - \phi \quad (14.1)$$

where ϕ , the *work function*, is a constant whose value depends on the nature of the surface. Einstein derived this result by postulating that the light beam consists of photons, each of energy hf . An electron inside the metal absorbs all the energy of a photon and may escape from the metal after losing at least an amount of energy ϕ in the process. The work function ϕ is the binding energy of the most energetic electrons in the metal. Energy conservation then requires that the maximum kinetic energy E of the ejected electrons is given by the photoelectric equation. This equation was verified in detail by Millikan in 1916.

The *stopping potential* V is the value of the retarding potential difference that is just sufficient to stop the *most energetic* photoelectrons emitted. Therefore the product of the stopping potential and the electronic charge, eV , is equal to the maximum kinetic energy that an emitted electron can have, i.e.

$$E = hf - \phi = eV. \quad (14.2)$$

Since

$$f = c/\lambda, \quad (14.3)$$

it follows that

$$eV = hc/\lambda - \phi. \quad (14.4)$$

14.3 Experiment 9 – photoelectric effect

The light source is a filament lamp operated from the mains. Radiation of different wavelengths is obtained by placing transparencies A (figure 14.2), each transmitting only a narrow band of wavelengths between the light source and the window of the photo tube.

The wavelength that you have to use in your calculations, is the midvalue of the wavelength band indicated in table 14.1.

Table 14.1 Midvalues λ of the wavelength band of the transparencies for experiment 9.

Transparency number	$\lambda(\text{nm})$
1	410
2	450
3	465
4	515
5	525
6	540
7	580
8	600
9	625
10	640
11	650

In this experiment you will not measure the photoelectric current. However, you will use the power dissipation in the filament of the lamp to determine when the photoelectric current is zero.

The mains current is given by

$$I = I_0 \cos \omega t$$

where $\omega = 2\pi f$ and f is the frequency (50 Hz). The power dissipated in the filament with resistance R , is

$$\begin{aligned}
 P &= I^2 R \\
 &= I_0^2 R \cos^2 \omega t \\
 &= I_0^2 R (1 + \cos 2\omega t) / 2.
 \end{aligned}
 \tag{14.5}$$

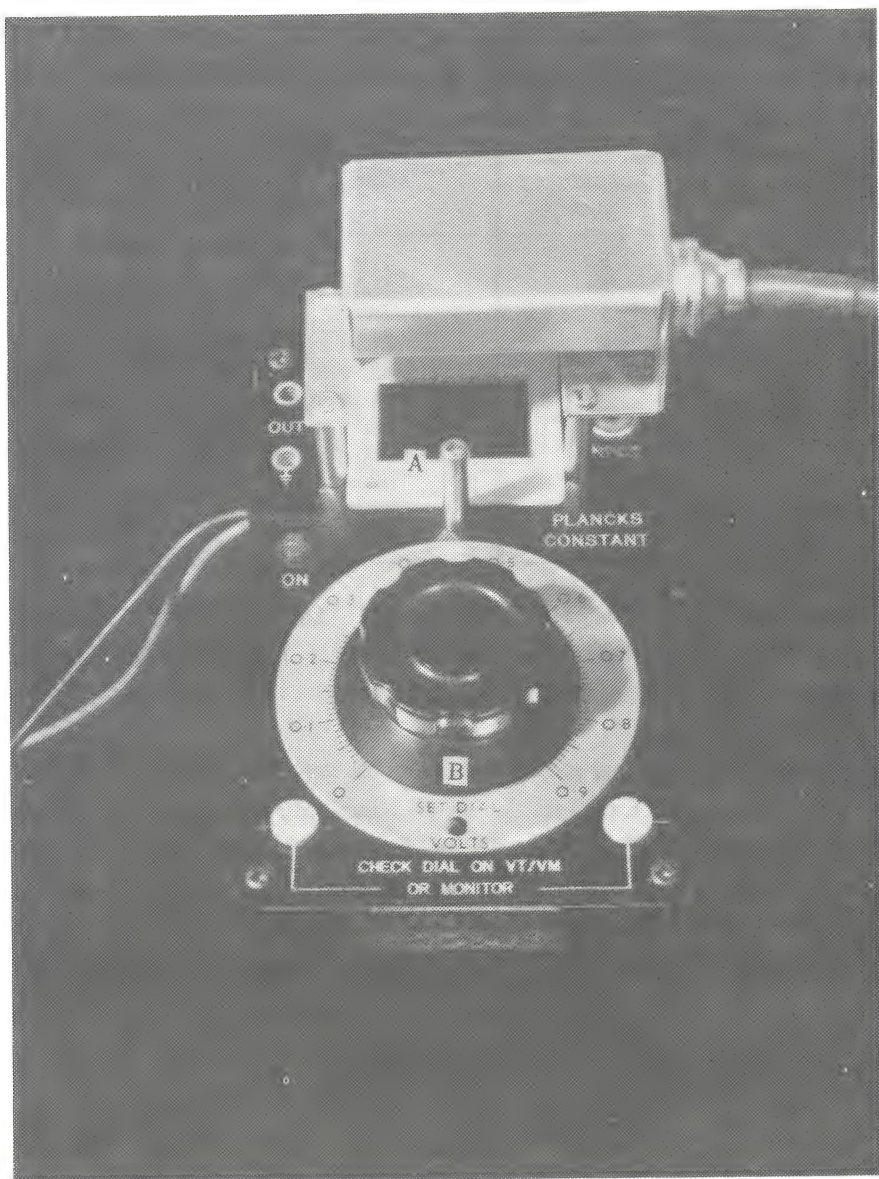


Figure 14.2 Apparatus for the photoelectric effect.

It is obvious that the frequency of the power versus time graph is 100 Hz. Since the photoelectric current is directly proportional to the intensity of the light, it will show a variation which has a 100 Hz frequency. This ripple is amplified and displayed on a cathode ray oscilloscope (CRO). Adjustment of the retarding (stopping) voltage V applied to the anode of the tube alters the CRO trace. Disappearance of the ripple on the CRO means that no current flows through the tube. In this way the stopping potential can be determined as a function of wavelength.

This experiment is limited by the nature of commercially available photocells. Heat from the lamp is one factor affecting it. Another factor is that some of the cathode material (which is evaporated on to the cathode) is deposited on the anode as well, so that when the potentiometer (B in figure 14.2) is turned beyond the cut-off point, the CRO trace reappears. This cut-off voltage will be less than the true value. In spite of these limitations, this experiment gives a fine illustration of how Planck's constant h is measured and will give the correct order of magnitude for h .

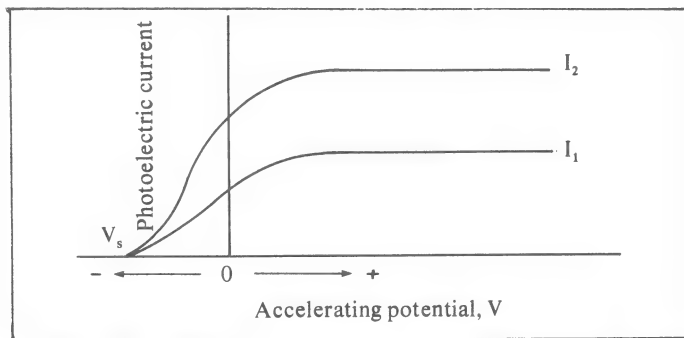
The experiment includes the following:

1. Determine the stopping potential for each of the transparencies at least 5 times.
2. Record the stopping potentials and corresponding wavelengths.
3. analyse the data statistically (refer back to chapters 1 and 2).
4. Do a linear regression analysis and draw a straight line graph. Indicate the errors obtained in step (3) on each data point by means of errorbars.
5. Determine Planck's constant, the work function of the metal, the threshold frequency and s_h and s_ϕ .

14.4 Questions

- 14.1 What is the essential conclusion of the photoelectric experiment?
- 14.2 On the wave picture, if the light is made dimmer, would you expect electrons to take longer before they are ejected? Explain. Experimentally electrons are ejected in a very short time, of the order of 10^{-9} s.
- 14.3 How do you visualise the ejection of an electron by an oscillating electromagnetic wave? Consider the electric field vector $\vec{E}$ and the appropriate expression for the force on the electron.
- 14.4 According to the wave picture of light, how would the kinetic energy of the ejected electrons be affected if the light intensity is increased?
- 14.5 What is so striking about the existence of a threshold frequency?
- 14.6 What is meant by threshold frequency?
- 14.7 How does one measure the maximum kinetic energy of the ejected electrons? Discuss only the principles that are involved.
- 14.8 On what single factor does the maximum kinetic energy of an ejected electron depend?
- 14.9 What is the only effect of an intensity increase?
- 14.10 How would you increase the intensity of the light beam in the experiment?

- 14.11 A graph of photoelectric current against accelerating potential between T and C in figure 14.1 is shown below.



- Why does the photoelectric current become independent of the accelerating potential if the latter is increased?
 - Why does the photoelectric current decrease as the accelerating potential is decreased?
 - Explain why a negative potential is required to stop the photoelectric current.
- 14.12 Draw a graph of the expected dependence of the photoelectric current on the light intensity. Explain. What will the effect of a different frequency be on the graph?
- 14.13 Give some suggestions for reducing the experimental error in the determination of Planck's constant.
- 14.14 Why do many electrons arrive outside the metal with a velocity less than the maximum value?

- 14.15 When a clear surface is irradiated with ultraviolet light it is found that no electrons are ejected from the surface unless the light has a wavelength less than 2.93×10^{-7} m.
- a) What is the work function (in eV) of the surface?
 - b) What is the maximum kinetic energy of the electrons that are ejected by light of wavelength 1.24×10^{-7} m?
- 14.16 How is the threshold frequency related to the work function?
- 14.17 Mention a few practical applications of photocells.
- 14.18 Compute the maximum velocity with which the electrons escape from the metal in question 14.15(b). Should relativistic equations be used for calculating the velocity? Explain.
- 14.19 Green light ejects electrons from a certain surface. Yellow light does not. Do you expect electrons to be ejected when the surface is illuminated with
- a) Red light? Why?
 - b) Violet light? Why?
- 14.20 What is the energy of a photon having a wavelength of 4×10^{-7} m?
- 14.21 What will be the change in stopping potential for photo-electrons emitted from the surface if a wavelength of the incident light is reduced from 4.0×10^{-7} m to 3.6×10^{-7} m?

- 14.22 Suppose that for light with a wavelength of 4.00×10^{-7} m the stopping potential is 2.00 volts, while for light with a wavelength of 6.00×10^{-7} m the stopping potential is 1.00 volt. Determine the value of Planck's constant and the work function of the surface.
- 14.23 Draw a graph of equation (14.5) to illustrate that the photoelectric current will show a 100 Hz ripple.
- 14.24 A photoelectric experiment yielded the following results:

Wavelength range of transparency (10^{-7} m)	Stopping potential (V)
0.8-1.0	12.4
1-2	7.5
2-4	3.0
3-5	2.2
5-7	1.1
7-9	0.57
9-11	0.20

Draw a straight line graph and do a linear regression analysis of the data to determine Planck's constant, the threshold frequency and the stopping potential of the anode. Determine the errors on both h and ϕ .

14.25 A photoelectric experiment yielded the following results:

Wavelength range of transparency (m)	Stopping potential (V)
(8-10 $\times 10^{-8}$)	13.5
(1-2 $\times 10^{-7}$)	7.4
(2-4 $\times 10^{-7}$)	3.4
(3-5 $\times 10^{-7}$)	2.8
(5-7 $\times 10^{-7}$)	1.7
(7-9 $\times 10^{-7}$)	0.98
(9-11 $\times 10^{-7}$)	0.60

Draw a straight line graph and do a linear regression analysis of the data to determine Planck's constant, the threshold frequency and the stopping potential of the anode. Determine the errors on both h and ϕ .

14.26 A photoelectric experiment yielded the following results:

Wavelength range of transparency (m)	Stopping potential (V)
(8-10 $\times 10^{-8}$)	13.8
(1-2 $\times 10^{-7}$)	8.2
(2-4 $\times 10^{-7}$)	4.0
(3-5 $\times 10^{-7}$)	2.7
(5-7 $\times 10^{-7}$)	1.9
(7-9 $\times 10^{-7}$)	1.2
(9-11 $\times 10^{-7}$)	0.9

Draw a straight line graph and do a linear regression analysis of the data to determine Planck's constant, the threshold frequency and the stopping potential of the anode. Determine the errors on both h and ϕ .

Appendices

Appendix A

Exponents and Logarithms

For real numbers:

$$a^n = a \times a \times a \dots n \text{ times}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{1/n} = \sqrt[n]{a}$$

$$a^{1/\infty} = \sqrt[\infty]{a} = a^0 = 1$$

$$(ab)^p = a^p \times b^p$$

$$a^p \times a^q = a^{p+q}$$

$$(a^p) \div (a^q) = a^{p-q}$$

$$a^{p/q} = \sqrt[q]{a^p} = (\sqrt[q]{a})^p$$

$$(a^p)^{q_1} = a^{pq}$$

If $a > 0$ and $a^n = b$ then $b > 0$ for all real values of n . Then $n = \log_a b$ and we say n is the logarithm of b to the base a ; i.e. n is the exponent to which the base a must be raised to give b . We can also say $b = \text{antilog}_a n$.

For calculations logarithms to the base 10 are used frequently. Usually the base is not indicated in this case. Therefore if

$$r = \log s, \text{ then } s = 10^r.$$

Logarithms to the base e are called natural logarithms.

If $y = \log_e b = \ln b$, then $b = e^y$. The notation $y = \ln b$ (pronounced "lin b") is generally accepted for natural logarithms. With p , q and a positive numbers and $a \neq 1$

$$\log_a p^n = n \log_a p$$

$$\log_a \sqrt[n]{p} = \frac{1}{n} \log_a p$$

$$\log_a (1/p) = -\log_a p$$

$$\log_a 1 = 0$$

$$\log_a (p \times q) = \log_a p + \log_a q$$

$$\log_a (p/q) = \log_a p - \log_a q$$

Also with $b \neq 1$ and $b > 0$

$$\log_a p = \log_b b = \frac{\log_b p}{\log_b a}$$

Appendix B

The Greek alphabet

A α alpha

B β beta

Γ γ gamma

Δ δ delta

E ϵ epsilon

Z ζ zeta

H η eta

Θ θ theta

I ι iota

K κ kappa

Λ λ lambda

M μ mu

N ν nu

Ξ ξ xi

O o omicron

Π π pi

P ρ rho

Σ σ sigma

T τ tau

Y υ upsilon

Φ ϕ phi

X χ chi

Ψ ψ psi

Ω ω omega

Appendix C

SI-Units

Quantity	Name of Unit	Symbol
length	metre	m
	millimetre	mm
	kilometre	km
mass	kilogram	kg
	gram	g
time	second	s
force	newton	N
work, energy	joule	J
power	watt	W
electric charge	coulomb	C
electric potential	volt	V
capacitance	farad	F
resistance	ohm	Ω
magnetic flux	weber	Wb
magnetic flux density	tesla	$T(\equiv \text{Wb m}^{-2})$
inductance	henry	H
area	square metre	m^2
frequency	hertz	Hz
density	kilogram per cubic metre	kg m^{-3}
velocity	metre per second	m s^{-1}
pressure	newton per square metre	N m^{-2}
moment	newton metre	N m
moment of inertia	kilogram metre square	kg m^2
temperature	kelvin	K
	degree celsius	$^{\circ}\text{C}$
light intensity	candela	cd

Appendix D

D1 Physical constants

Speed of light	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$
Gas constant	$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Avogadro's number	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Coulomb's constant	$k = 9.00 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$
Boltzmann's constant	$k_B = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Planck's constant	$h = 6.63 \times 10^{-34} \text{ J s}$
Electron charge (magnitude)	$e = 1.60 \times 10^{-19} \text{ C}$
Electron mass	$m_e = 9.11 \times 10^{-31} \text{ kg}$
Proton mass	$m_p = 1.67 \times 10^{-27} \text{ kg}$
Bohr radius	$a_0 = 5.29 \times 10^{-11} \text{ m}$

D2 Conversion factors

$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$
$1 \text{ \AA} = 10^{-10} \text{ m}$
$1 \text{ inch} = 25.4 \text{ mm}$
$1 \text{ cal} = 4.19 \text{ J}$
$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$
$180^\circ = \pi \text{ rad}$

Appendix E

Linear regression analysis

During the laboratory course a number of IBM personal computers will be available for the linear regression analyses that have to be done. One of these computers is shown in figure E.1. You communicate with the computer by typing commands on the keyboard and the computer communicates with you by asking questions, giving commands or listing results on the screen of the monitor.

A detailed description of the procedure to be followed to do a linear regression analysis of a data set will be given when you attend the laboratory course in Pretoria.



Figure E.1 IBM personal computer.

Appendix F

Demonstration experiments

F1 Introduction

In the corridor of the Physics Department where students move around daily during the laboratory course, a number of demonstration experiments have been installed. Next to each experiment a short review on the applicable theory is displayed in the form of a poster. The experiments are very instructive and we want to encourage you to take the time to have a look at them. A short summary of the different demonstration experiments follows below.

F2 Colliding balls

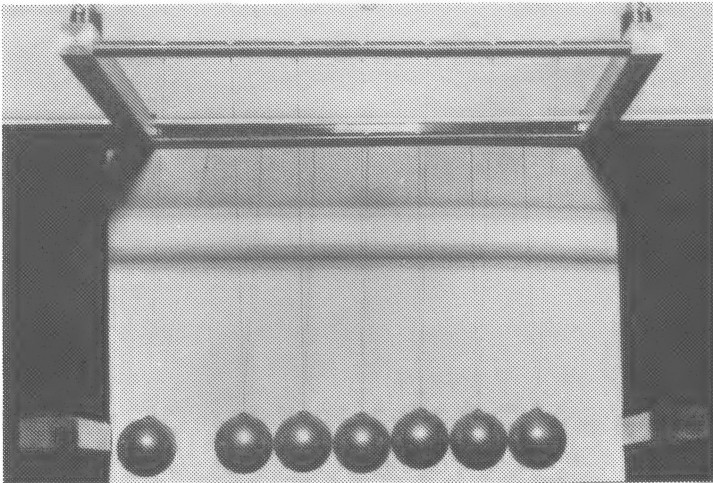


Figure F.1

The apparatus is known to most of us in the form of an ornament which usually stimulates thought. The outstanding characteristic of the apparatus is of course its ability to count! If n balls are incident from the left, n balls are shot out to the right. Why? The phenomenon can be explained on different levels, depending on the assumptions that are made. You should get good practice in applying the laws of conservation of energy and momentum by trying to find an explanation by making use of the following three assumptions:

1. All motion takes place along a straight line.
2. The collisions are elastic.
3. All the balls in motion after a collision are moving together, i.e. they all have the same velocity.

The phenomenon can of course also be explained along the lines of repeated collisions and pressure wave propagation

F3 The Brachistochrone

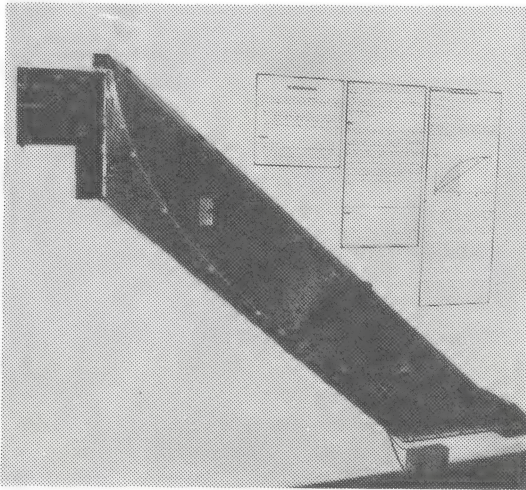


Figure F.2

This experiment will provide you with the opportunity of observing something which will make the reliability of your own eyes doubtful! Predict intuitively which of the two steel balls of the apparatus, depicted in Figure F.2, will arrive at the bottom first – the one which rolls down along the straight track, or the one which follows the curved track? Now perform the experiment according to the instructions on the poster.

Although it is difficult to believe, this experiment proves that the shortest "distance" between A and B in a gravitational field is a curve and not a straight line.

Why does the ball on the curved track reach the bottom first? It boils down to the fact that during its steep descent along the first part of the track, it attains a great enough velocity to cover a greater distance in a shorter time. To understand this statement better, two graphs are shown next to the apparatus, showing velocity as a function of time for the two tracks. In principle it means that the time saved along the curved track is a compromise between the increase in velocity and the longer path length. The shape of the track which required the shortest travel time from A to B is known as the **brachistochrone**.

F4 The Foucault pendulum

The Foucault pendulum is driven electromagnetically in such a way that the amplitude of the pendulum remains constant. The Foucault pendulum illustrates the rotation of the earth – in other words the fact that the earth is not an inertial frame of reference. This demonstration was performed for the first time by Jean Foucault in Paris in 1851, when he suspended a mass of 28 kg from a wire of 67 m from the dome of Les Invalides so that it could swing in any direction. The period for a pendulum of this length is approximately 16,4 s. Beneath the pendulum a circular mound of sand was placed in such a way that a metal point, extending downwards from the

pendulum, marked the sand so that it was clearly visible that after an hour the plane in which the pendulum swung had shifted by more than 11°



Figure F.3

Why does the plane of the pendulum rotate? If the Foucault experiment were executed at the North Pole, we would observe that the plane of the pendulum remains fixed in an inertial frame while the earth rotated once in 24 hours beneath the pendulum. Because the earth rotates in an anti-clockwise direction, an observer at the North Pole will see the plane of the pendulum rotating clockwise relative to himself.

The problem is rather more difficult to analyse when we move away from the North Pole.

Appendix G

Laboratory course

14 to 25 August, 1995

G1 Introduction

This appendix contains important information concerning the laboratory course. Please study it carefully to avoid unnecessary questions. *Please note that this Appendix refers only to students who will be admitted to the laboratory course. Students will be informed by telegram whether they have qualified for admission immediately after the third assignment has been marked.*

G2 General information

1. The course consists of the following:
 - Execution of experiments and the writing of reports (paragraph G3).
 - Short tests on the theoretical and practical aspects of the experiments (paragraph G5).
 - A written examination on the last Friday of the laboratory course which will cover all these aspects (paragraph G4).
2. The time table for every day, *except* the *first* Monday and the *last* Friday, is as follows:

- 08h30 – ^{08h45}~~09h00~~ A lecturer discusses the experiment that you will be executing during the day in detail in the laboratory.

- ^{08h45 – 09h00} 09h00 – 09h15 A short test will be written in the laboratory on theoretical and practical aspects of the experiment that you will be executing during the day.

- ^{09h00} 09h15 – 12h15 Execution of the experiment in the laboratory.
- ^{12h15 – 13h00} 12h15 – 13h00 Lunch

- 12h15 – 15h45 Writing of the report. You have to submit your report at the end of this period.

- ^{16h00} 16h00 All reports must be handed in for marking at 16h00

3. The program for the first Monday is as follows: (Monday, 14th August)
Exam. Hall 2, TUV

- ^{16h00} 08h30 – 15h45 All students attend an orientation lecture in which procedures to be followed during the laboratory course will be discussed.

4. Maps of the campus and the laboratory building as well as a street map of Pretoria are included to enable you to locate the various laboratories, offices, the seminar room and the cafeteria.

5. **SMOKING IS NOT ALLOWED** in the laboratory building and the seminar room.

G3 Execution of experiments

1. Information on the experiments, laboratories and the lecturers responsible will be given in a special Tutorial Letter which you will receive before the laboratory course starts.

2. Chapters 1, 2, 3 and 4 of the Study Guide have to be studied *before* the laboratory course starts. Please pay special attention to paragraph 3.6. (Hints on how to plot graphs) and Chapter 4 (Planning an experiment and writing a report).
3. The time table for the execution of the experiments will be available before the laboratory course starts. Each student will be given a number and will do the experiments according to the sequence corresponding to this number.
4. The lecturers will be available to assist you and will also use the opportunity to discuss the theoretical and practical aspects of the experiments.
5. Since the time available for performing an experiment and writing a report is limited, it is imperative that you are well-prepared for the course. Please note that you will write a short test on all aspects of the experiment *before* you attempt the experiment.
6. Experiments are to be performed independently as far as possible. The rule concerning the independent treatment and interpretation of results will be rigidly enforced.
7. Examination books and graph paper for the writing of reports will be supplied by the University. It is *not* necessary to buy a laboratory book.
8. All measurements must be written *immediately, directly* and *in ink* in an examination book. The measurements must be initialed by the lecturer before you leave the laboratory.

9. Start to process your results immediately after the completion of the practical part of the experiment. Microcomputers are available for the linear regression analyses of data. Assistants will be available between 14h00 and 15h45 to help you with any problems that you may encounter. Write a short report which is accurate, complete and clear.
10. The completed report is submitted to the assistants before 15h45 on the day that the experiment was executed (excluding the first Monday). During the course of the next morning the lecturer will discuss your report in your presence in the laboratory. Your marked report will be kept by the Department.

G4 Written examination

1. You will sit for the written examination on the last Friday from 14h00 until 16h00.
2. This examination will cover all aspects of the course.

G5 Short tests

These tests consist entirely of multiple choice questions which you have to answer on the question sheet by selecting and circling the most correct of the given numbered alternatives.

Example: say question 1 of the test is as follows:

1. A SPACE SHUTTLE IS ORBITING THE EARTH IN A UNIFORM CIRCULAR MOTION.

A) THERE ARE NO FORCES ACTING ON THE SHUTTLE.

- B) THE SHUTTLE IS BEING ACCELERATED TOWARDS THE CENTRE OF THE EARTH.
- C) THE SHUTTLE IS BEING ACCELERATED AWAY FROM THE EARTH.
- D) THE SPEED OF THE SHUTTLE IS CONSTANT.
- E) THE VELOCITY OF THE SHUTTLE IS CONSTANT.

1:A) 2:B) 3:B)+D) 4:C) 5:D) 6:E) 7: NONE OF THESE.

Select and circle the *most* correct of the given numbered alternatives.

The given example could be marked as follows:

1: -1 mark, 2: 1 mark, 3: 2 marks, 4: -1 mark, 5: 1 mark,

6: -1 mark, 7: 0 marks.

The most correct answer (3) earns 2/2 for the question.

The results of the tests will be displayed on a notice board next to laboratory 1-22.

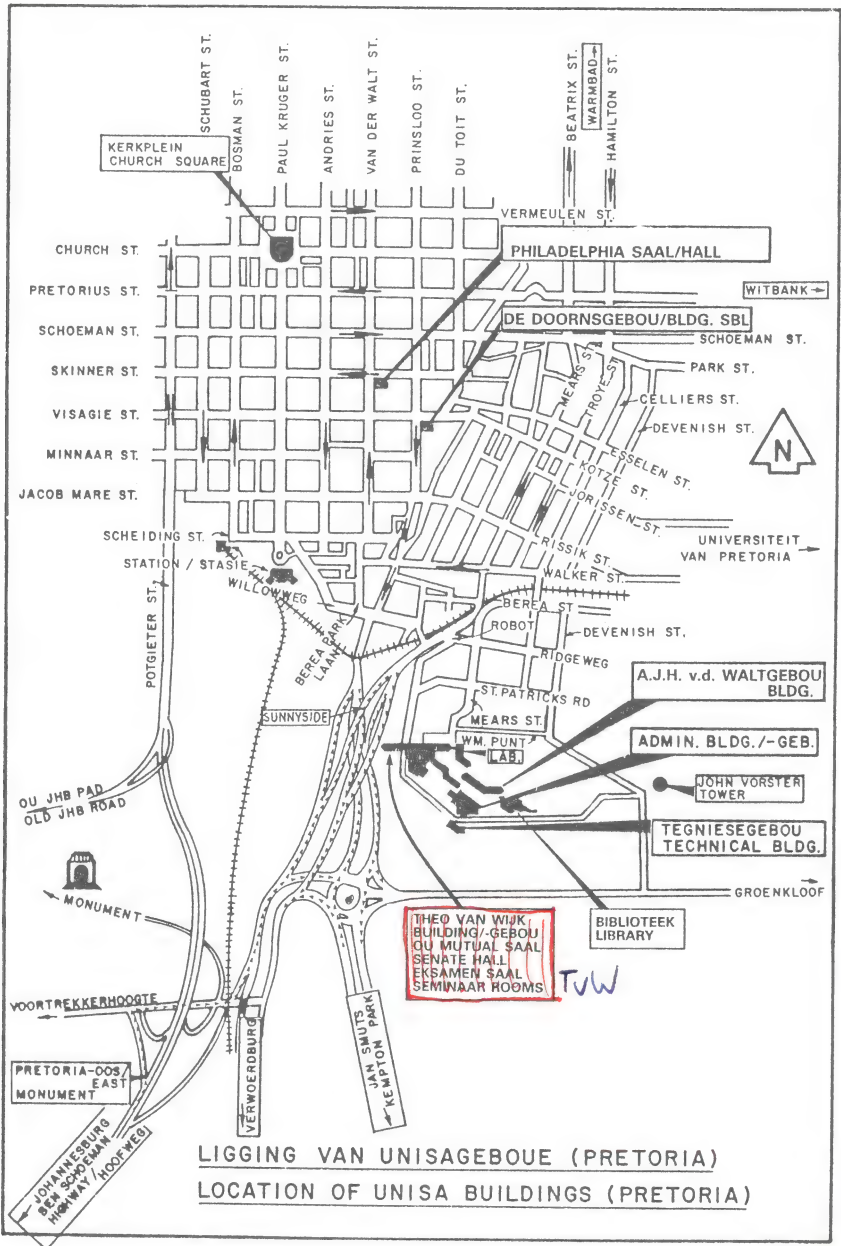
G6 Accommodation

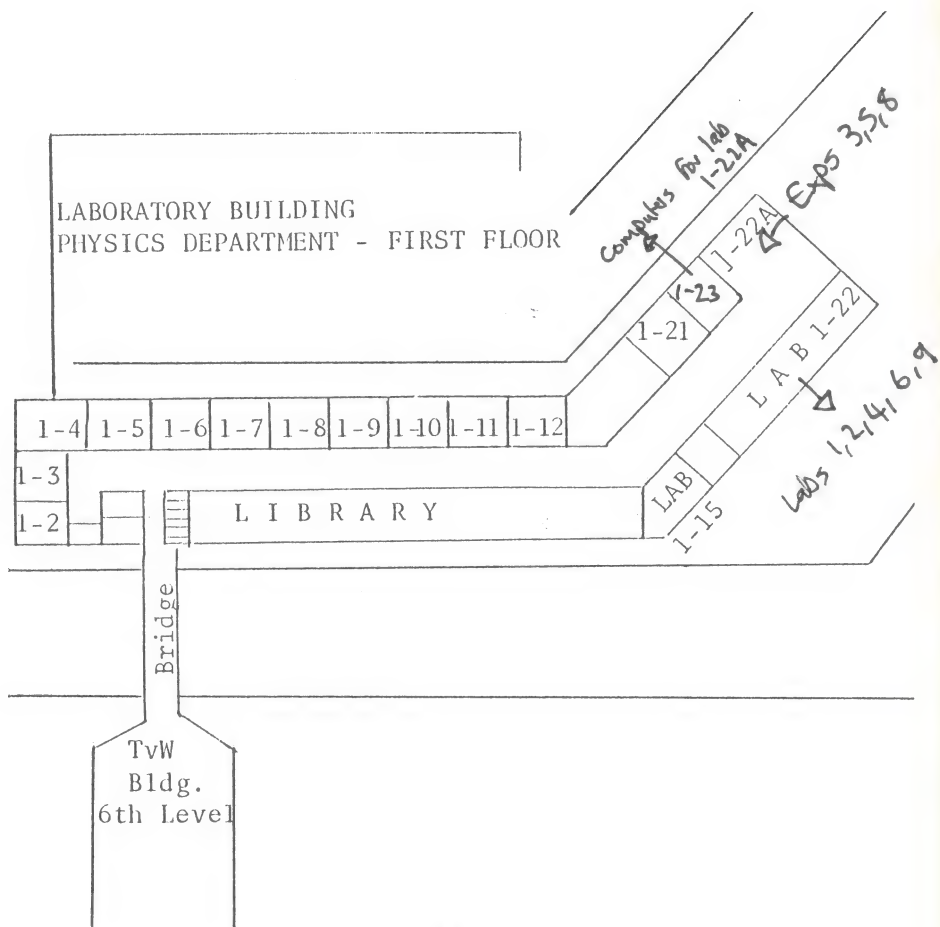
Students are responsible for all arrangements concerning their accommodation and transport during the course.

G7 Final mark for PHY 103-8

1.	Experimental work and writing of reports:	40%
2.	Short tests:	20%
3.	Written examination:	40%

G8 Maps





COMPUTER

Type : PHY103 ☐

Enter number of data points, (An ^{xy} x value are one datapoint)
(dots on graph)

Enter data : x ☐ ^{sp} y

ie. 1.1 ☐ 2.5E3

Exp	Test (1/12)	Reports (120)
1	6	219
2	10	
3	7	18
4	12	20
5	8	15
6	7	16 $\frac{1}{2}$
7	6	19
8	7	19
$\frac{63}{96} = \frac{65}{100} = \frac{13}{20}$		



$$b \sin \theta = n\lambda \quad N = 1, 3, 5, 7$$

$$N = 2, 6, 10, 14$$

$$\begin{aligned} \text{minima } N &= 1, 2, 3, 4, 5 \\ \text{maxima } N &= 1, 2, 3, 4, 5, 6, 7 \end{aligned}$$

$$\begin{aligned} \frac{1}{2} b \sin \theta &= (n - \frac{1}{2}) \lambda \\ b \sin \theta &= (n - 2) \lambda \end{aligned}$$

PHY103-8 PRACTICAL WORK

1995

1. Program

Monday 14 August: Lecture and introduction to practical
08:30 - Exam hall 2, TvW

Friday 25 August: Exam
12:00 - 5-54, 5-178 and 7-94, TvW

From Tuesday 15 August to Thursday 24 August the program for every day is as follows:

08:30 - 09:00	(0.50 h)	Discuss experiment of the day
09:00 - 09:15	(0.25 h)	Short test
09:15 - 12:00	(3.00 h)	Experiment
12:00 - 12:15	(0.25 h)	Discussion of experiment of previous day
12:15 - 13:00	(0.75 h)	Lunch
13:00 - 15:45	(2.75 h)	Report writing

2. Lecturers

Exp no.	Title	Lecturer	Office no.	Afternoon sessions
1	Linear air track	Dr P M Stoop	5-117	08-15 & 08-21
2	Circular motion	Mrs J A Cilliers	1-8	08-16 & 08-22
✓ 3	Simple harmonic motion	Mr J H Brink	1-6	08-17 & 08-23
✓ 4	Electrical measurements	Mr M L Lekala	5-123	08-18 & 08-24
✓ 5	e/m of electron	Mr G Vermaak	5-120	08-15 & 08-21
✓ 6	Current balance	Mr P Mjwara	1-10	08-16 & 08-22
✓ 8	Physical optics	Mrs L L Howell	1-5	08-17 & 08-23
✓ 9	Photoelectric effect	Miss R Rothner	5-125	08-18 & 08-24
	Computer tests	Mrs B Fenrèych		

Do at least
7 data points
for each class.

3. Experiments

Laboratory number	Date							
	08-15	08-16	08-17	08-18	08-21	08-22	08-23	08-24
001-010, 081, 089	1	2	3	4	5	6	8	9
011-020, 082, 090	9	1	2	3	4	5	6	8
021-030, 083, 091	8	9	1	2	3	4	5	6
031-040, 084, 092	6	8	9	1	2	3	4	5
041-050, 085, 093	5	6	8	9	1	2	3	4
051-060, 086, 094	4	5	6	8	9	1	2	3
061-070, 087, 095	3 ✓	4 ✓	5 ✓	6 ✓	8 ✓	9 ✓	1	2
071-080, 088, 096	2	3	4	5	6	8	9	1

Report Cover

Name :

Lab number :

Student Number :

4 Reports

4.1 Mark allocation for reports

1. Introduction

1.1 Aim

1.2 Theory

(2)

2. Method

(2)

3. Results

3.1 Tables

3.2 Graphs

3.3 Calculations

(12)

4. Discussion and conclusion

(4)

Could I improve the experiment?
[20] Am I entitled to say what I did

4.2 Error analysis

- For measurements repeated a number of times n adjusted standard deviation should be used

Adjusted σ

$$\sigma = \left[\frac{\sum \delta^2}{n-1} \right]^{1/2}$$

(used for small number of measurements)

This should be indicated by an error bar (drawn to scale) on graph.

- In the case of measurements taken once only, error analysis from linear regression is to be used.
- Combination of errors: On p54 in the study guide, a summary is given of the way in which the calculation is done when a combination of errors is taken into account. You will be expected to show in your report how you did these calculations.

~~the combination of errors (p54)~~

- show an example of how
you did one, and tabulate
the rest

4.3 Repeated measurements

	Repeated measurements
1. Air track	✓
2. Circular motion	×
3. Simple harmonic motion	✓
4. Electric measurements	×
5. e/m of electron	×
6. Magnetic force	×
7. Physical optics	✓
8. Photoelectric effect	✓

Take into account friction
(~~do~~ do calculation to take friction
Hint into account. See examples)

Report checklist

ACTIVITY	Experiment								
	1	2	3	4	5	6	7	8	9
Tables									
Multiplicative factor at top of columns in tables.									
Units in tables.									
Precision of the measurement reflected in the number of significant digits listed in the tables									
digits listed in the tables. (2 extra units are allowable for intermediate)									
~~Wk Scale~~ ~~units~~									
Graphs									
Plot one graph per page.									
Sensible scale.									
Descriptive labels for axes.									
Units at axes (if applicable also the multiplicative factor)									
Error bars.									
Heading.									
~~Wk Scale~~									
Presentation of final results									
Number of digits in final results not more than precision of experimental measurements									
umber of digits after decimal point in deviation (error) same as for the primary value.									
State the nature of the error calculated (ex. <u>standard deviation</u> , mean deviation, linear regression...)									
Conclusion									
Referring to the purpose of the experiment, make conclusions regarding ~~the~~									
success of experiment.									
Address all aspects of data in conclusions - especially data that looks suspicious.									
Justify any additional assumptions that had to be made.									
Motivate all statements.									
Possible suggestions to improve the results (if any).									
(Must?) ~~Can~~ make comments on <u>systematic</u> + <u>random</u> errors									
Style									
Write preferably in the third person - passive voice!									
Write aim and method in past tense.									
Introduce notation.									
Use diagrams to make explanations clear. ("One picture is worth a thousand words!!")									
In the case where calculation of a certian quantity was made a number of times, table the values and show only one calculation as an example of what was done									

$$\frac{E}{m} = \frac{2U}{E^2}$$

$$B = \left(\frac{6}{5}\right)^2 \cdot 4 \pi \times 10^{-7} \text{ N A}^{-2} \times \frac{N}{R}$$

$$R = \left(\frac{6}{5}\right)^2 \cdot 4 \pi \times 10^{-7} \frac{\text{N}}{\text{A}^2} \times N$$

$$B = \frac{\left(\frac{6}{5}\right)^2 \cdot 4 \pi \times 10^{-7} \text{ N} \times 30 \times 1}{0.06 \text{ m}}$$

$$= 4.23/4.78 \times 10^{-8} \text{ T}$$

$$\frac{1 \text{ kg}}{1 \text{ m}^3} = \frac{1}{1 \text{ m}^3}$$

EXPERIMENT 8

EXPERIMENTS IN PHYSICAL OPTICS

THEORY

You need to know the following:

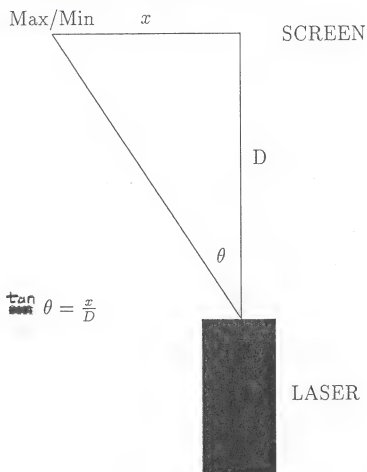
1. The difference between refraction, diffraction and interference.
2. Relationships between λ , θ and other variables in an experiment.
3. The definitions of coherent and monochromatic light.

1 Experiment 8(a): Single Slit Diffraction

$$b \sin \theta = N\lambda; \quad N = \pm 1, \pm 2, \pm 3, \dots$$

AIM:

To calculate an average value for the thickness of hair.



N	$2x$	x	θ	$\sin \theta$	b

2 Experiment 8(b): Diffraction pattern of blood cells

$$d \sin \theta = N\lambda; \quad N = 1.220, 2.233, 3.238, \dots \text{minima}$$

AIM:

To calculate the diameter of a red blood cell.

3 Experiment 8(c): Diffraction by a grating

$$a \sin \theta = N\lambda; \quad N = 0, \pm 1, \pm 2, \dots \text{maxima}$$

$$\sin \theta = \left(\frac{\lambda}{a}\right) N$$

$$d = a \sin \theta$$

$$\Delta \lambda = a \Delta \sin \theta$$

AIM:

To determine the wavelength of the laser beam.

In order to calculate a , the above equation has to be rearranged into the form

$$y = mx + c:$$

.....

and vs must be plotted.

The slope gives

N	$2x$	x	$\tan \theta$	θ	$\sin \theta$

4 Experiment 8(d): Electron diffraction

$$\lambda = \frac{h}{(2meV)^{\frac{1}{2}}}$$

$$2d \sin \theta = N\lambda; \quad N = 0, 1, 2, 3, \dots \text{maxima}$$

AIM:

To calculate the average distance between the planes in a crystal.

3000

$$D = 15 \text{ cm}$$

$$N_1 \quad d(2x) = 1.6 \text{ cm} + 1.5 \text{ cm}$$

$$N_2 \quad d_2(2x) = 30 \text{ cm} + 2.9 \text{ cm}$$

$$d_3(2x):$$





UNISA

UNIVERSITY OF SOUTH AFRICA



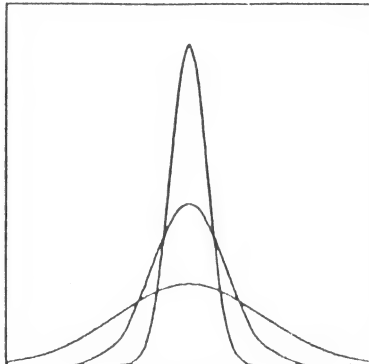
96167793

DEPARTMENT OF PHYSICS

Physics

PHY103-8 (Practical work)

Tutorial Letter 101 – 1995



Contents

- 1 Introduction**
- 2 Tutorial matter**
- 3 Communicating with Unisa**
 - 3.1 Letters
 - 3.2 Telephone calls
 - 3.3 Appointments
- 4 Assignments**
 - 4.1 Preparation of assignment covers
 - 4.2 Key to assignment problems
 - 4.3 Graph paper
 - 4.4 Due dates
 - 4.5 Correction of assignments
 - 4.6 Admission credit
- 5 Numbering of tutorial letters**
- 6 Examinations**
 - 6.1 Composition of examination papers
 - 6.2 Calculations and the use of pocket calculators
 - 6.3 Memorisation of formulae
- 7 National service**
- 8 Module PHY103-8: Practical course – Specific details and assignments**
 - 8.1 Tutorial matter
 - 8.2 The laboratory course and examination
 - 8.3 Entrance requirements to laboratory course
 - 8.4 Exemption from Practical work
 - 8.5 Assignments



1 Introduction

On behalf of the Department of Physics I extend a cordial welcome to you. I hope you will find your studies interesting and rewarding.

This tutorial letter contains important information in connection with the prescribed work for the module, summaries of assignments, work programs, entrance requirements to the examinations, etc. You are asked to study the contents of this tutorial letter carefully. **Students regularly make enquiries about matters which are fully discussed here.** Additional information may be found in the Faculty Brochure, Parts 1 and 6 of the Calendar and the brochure **Unisa: Services and Procedures** for 1995.



2 Tutorial matter

The tutorial matter for this module consists of the following:

- 1 study guide
- 1 cassette (on registering)
- a number of tutorial letters which you will receive during the course of the year.

There are no prescribed textbooks for PHY103-8.

N.B. The abovementioned tutorial matter will *not necessarily* be available at the time of registration.

When you enrol you will receive, in addition to the available tutorial matter, an **Inventory for the current academic year** which lists only the items available from the Department of Despatch at that stage.

Check the tutorial matter you have received against this inventory. You should have received all the items specified in the inventory unless there is an explicit statement to the contrary (e.g. out of stock). If any of the items is missing, follow without delay the instructions on the back of the inventory.

Please note! Use only the telephone number in the **Inventory** for future enquiries about missing tutorial matter.



3 Communicating with Unisa

You are invited to discuss any aspect of the work with the lecturer concerned by means of a letter, a telephone call or a pre-arranged visit. Please pay attention to the procedure to be followed in each case.

3.1 Letters

Students must address all correspondence to

The Registrar (Academic)
UNISA
P O Box 392
PRETORIA
0001

You may enclose more than one letter in the same envelope. Do not however address several departments in one letter. This will undoubtedly cause delay. Write a separate letter to each department and mark each letter clearly for the attention of the relevant department.

When writing to Unisa, **always** mention your student number, the subject and the module code at the top of the letter. Remember to have your student number ready whenever you contact the University.

3.2 Telephone calls

We supply the following important telephone numbers for your convenience (the dialing code for Pretoria is 012):

Prof EC Reynhardt (Head)	429-8062
Prof H Fiedeldey	429-8027
Prof WS Verwoerd	429-8013
Prof SA Sofianos	429-8014
Mr JH Brink	429-8005
Mr C J Terblanche	429-8029
Dr I Basson	429-8026
Dr CA Engelbrecht	429-8031
Mrs JA Cilliers	429-8018
Mrs LL Howell	429-8478
Mr P Mjwara	429-8302
Mr G Vermaak	429-8030

If you are unsuccessful in contacting a specific lecturer at his or her personal telephone number, you are advised to contact our departmental secretary at 429-8027 and ask her to locate the lecturer. Telephone calls should be made during the official office hours, i.e. from 07:45 to 13:00.

Telephone enquiries about administrative matters should be addressed to the relevant section. Consult the brochure **Unisa: Services and procedures** in this regard.

3.3 Appointments

Lecturers are available for pre-arranged appointments on weekdays from 07:45 to 16:00.



4 Assignments

The details of the assignments for this module are given in section 8.5 of this tutorial letter. You are advised to work through the relevant chapters in the study guide thoroughly before attempting to solve assignment problems. It is essential to start working on an assignment **at least** one month before the due date.

4.1 Preparation of assignment covers

Please follow the instructions on the assignment cover. Remember that your assignment must have precisely the same number as the one specified in this tutorial letter. Even though assignment 2 may be the first assignment done by you, it must be numbered 02 and not 01.

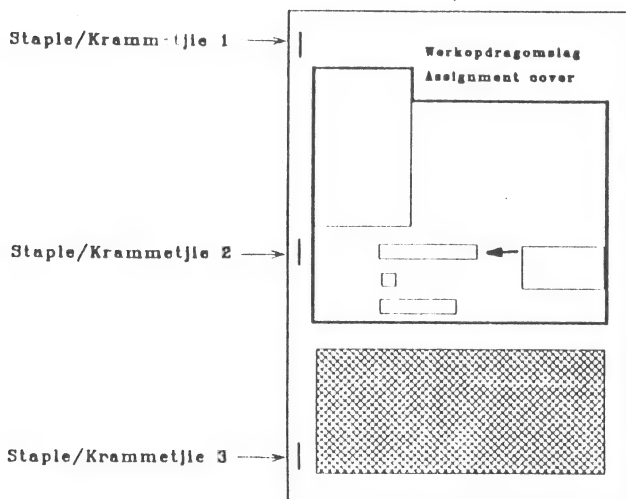
Specify the module code and assignment number in all enquiries about assignments.

IMPORTANT: Under no circumstances should you submit assignments by facsimile machine. Unisa will not accept assignments sent in this manner.

Please present the solutions to the problems for an assignment in the order indicated in the tutorial letter, so as to facilitate correcting.

We have a special request concerning your Physics assignments. It will make it easier to handle your assignment if you follow these instructions:

1. Staple the assignment cover at 3 places as indicated in the diagram.



2. Write the problem numbers in the middle of the page and do not use the ± 1 cm space to the left of the left margin for writing. Use this space for stapling as requested above. If you use graph paper, also allow a ± 1 cm margin for stapling.
3. Please separate the sheets at the top so that we can page them after they have been stapled.

Thank you for your cooperation.

Enquiries about assignments (e.g. whether or not the University has received an assignment, marks, when it was returned, etc.) may be addressed to:

Assignments Section: Telephone (012) 429-4144

4.2 Key to assignment problems

A tutorial letter (the so-called *key*) containing solutions to all the assignment problems will be mailed to you soon after the due date.

A post mortem letter containing comments and advice on the specific assignment, is sent out after the assignment has been marked. Read it attentively.

4.3 Graph paper

For most of the assignments you will be required to plot a number of graphs. For this purpose, you should use the GRAPH PAPER which will be provided with the rest of your study material for this module. Please plot only one graph per page!

4.4 Due dates

As mentioned above, solutions to the assignment problems are sent to students after their assignments have been marked. Therefore, an assignment that reaches the registrar after the due date might not be corrected and no admission credits will be awarded.

To ensure that your assignment arrives on time, you should mail it about *ten days before the due date*. If you live very far from Pretoria or in an isolated area, you should mail your assignment even earlier.

If you are unable to complete the assignment on time for whatever reason, you are advised to submit the partly completed assignment on or before the due date.

4.5 Correction of assignments

The correcting of assignments starts on the due date regardless of whether an assignment arrived three months or three days before the due date. You can expect your assignment to reach you about three weeks after the due date.

4.6 Admission credit

Unisa's computer is programmed to automatically admit a student to the examination provided the student has gained at least 100 examination admission credit marks for the module. The maximum admission credit marks that can be obtained for each assignment are listed in section 8 of this tutorial letter.

The marks obtained in the assignments have no direct effect on the final mark obtained by a student.

5 Numbering of tutorial letters

The numbering of tutorial letters is explained in the brochure **Unisa: Services and procedures** for 1995.

6 Examinations

6.1 Composition of examination paper

The examination paper for PHY103-8 will evaluate your knowledge and comprehension of the fundamental concepts of experimental work (as discussed in chapters 1-5 of the study guide). In addition, there will be questions on the individual experiments (chapters 6-14 of the study guide), as well as problems similar to those set for the assignments.

6.2 Calculations and the use of pocket calculators

Since the numerical solution of problems is an important part of the modules in Physics, you are strongly advised to buy a pocket calculator, which may be used during the practical course and in the examinations if it complies with the requirements and restrictions which are listed in part 6 of the Calendar.

6.3 Memorisation of formulae

You are definitely not expected to memorise all the formulae in the study guide. Since not all equations needed in the examination will be given in the paper, you should, however, know by heart **all the basic laws and definitions**. You should be able to derive all the formulae yourself.

7 National service

Admission to the examination of students doing military service is discussed in part 1 of the Calendar.

Please note that this year's PHY103-8 laboratory course is offered **only from 31 July to 11 August**, at Unisa.

8 Module PHY103-8: Practical work

8.1 Tutorial matter: *study guide*

The syllabus is covered by the study guide.

Use the prescribed textbooks of the other first year modules to look up concepts which you don't understand or which are unclear.

8.2 The laboratory course and examination

Students registered for this module must attend a laboratory course of 10 days (2× Monday to Friday), to be conducted in the *Unisa Physics Laboratory* from **31 July to 11 August**. The course not only comprises the execution of the prescribed experiments, but also practical demonstrations, lectures and tutorial sessions on practical and theoretical aspects of all first level modules. The only *examination* for this module is conducted on the *last day of the laboratory course* and the final mark will be available soon afterwards. The final mark is based on the marks obtained for the practical work, the tests written during the course and the examination.

8.3 Entrance requirements to laboratory course

To gain admission to the laboratory course, a minimum admission credit of 100 must be attained for assignments 1, 2, 3 and 4 together.

All students registered *before* 1 February 1995 must submit all 4 assignments. Students whose registration is delayed until after 1 February, must submit assignments 2, 3 and 4 and ensure that they catch up with the work covered by assignment 1.

You will be notified by telegram soon after the 1st of July whether you have been admitted to the course in August at Unisa.

Further details concerning this course are given in the appendix of the PHY103 study guide.

8.4 Exemption from Practical work

Exemption from practical work and the practical module may, with the approval of the Head of the Department, be granted to students who have **previously** completed the practical course concerned with satisfactory results at a recognised university.

For this purpose the student must supply the Head of the Department with a certificate, issued by the university concerned, in which they stipulate

- a) whether the course concerned is recognized by that university for admission to the next higher level of the pure BSc curriculum in Physics
- b) the final mark awarded for the student's practical work, and the year in which it was obtained.

If exemption is granted, the mark obtained will be regarded as the final examination mark. If a student has taken a complete Physics I course at another university but passed only the practical work, exemption from PHY103 may be considered on condition that three other first-level modules in Physics are passed within two years after exemption from PHY103 has been granted. Exemption from PHY103 will not be considered if more than **five years** have elapsed since the student passed the practical work.

Only those students who have studied **full-time** at a **recognised university** will be considered. Students are requested **not to make arrangements** to do the practical work at another university **while registered for the practical module at Unisa**. In such instances exemption from the practical module will not be granted.

8.5 Assignments

Problems are taken from the PHY103-8 Study guide.

Assign- ment no.	Due date	Chap- ters	Problems to be submitted	Maximum admission credit
01	15 February	1 2 3 4 5	1.8, 1.14 2.1, 2.6, 2.10 3.1, 3.10 - 5.4, 5.8	40
02	22 March	6 7 8	6.8, 6.14, 6.19 7.5, 7.12, 7.19 8.2, 8.16, 8.23	50
03	17 May	9 10 11	9.2, 9.19, 9.21 10.4, 10.12, 10.23 11.5, 11.14, 11.16	70
04	21 June	12 13 14	12.3, 12.10, 12.26 13.4, 13.15, 13.18 14.6, 14.18, 14.24	90

PROF EC REYNHARDT
HEAD: DEPARTMENT OF PHYSICS



UNISA UNIVERSITY OF SOUTH AFRICA



96158352

DEPARTMENT OF PHYSICS

PHYSICS PHY103-8

TUTORIAL LETTER 201 – 1995

KEY FOR ASSIGNMENT 1

1.8

Relative frequency is calculated from

$$r_i = \frac{f_i}{\Sigma f_i}$$

for the given data, to render the values tabulated below:

x	f	r
15	1	0.02
16	4	0.07
17	7	0.12
18	12	0.20
19	13	0.22
20	9	0.15
21	6	0.10
22	4	0.07
23	3	0.05
24	1	0.02

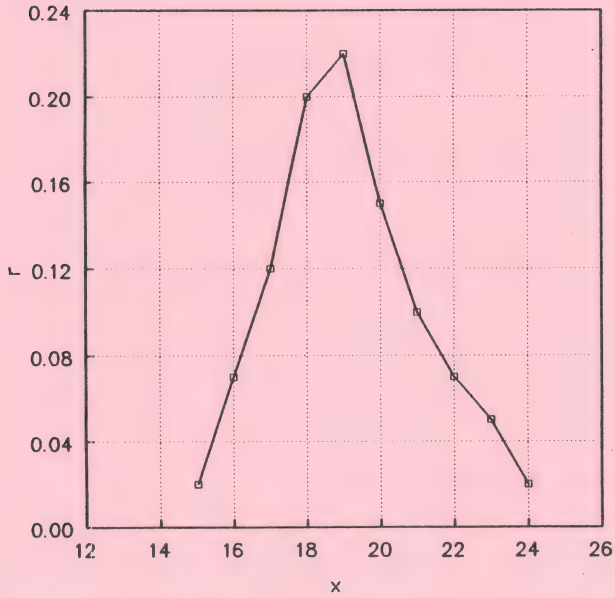


Figure 1: Relative frequency distribution.

1.14

(a) Let

$$D = kd$$

where d is the diameter of the spores in cm, and $k = 5880$. The mean value of D is given by

$$\begin{aligned}\bar{D} &= \frac{\sum_{i=1}^{10} f_i D_i}{\sum_{i=1}^{10} D_i} \\ &= \frac{3766 \text{ cm}}{200} \\ &= 18.83 \text{ cm.}\end{aligned}$$

The mean diameter of the spores is therefore

$$\begin{aligned}\bar{d} &= \frac{\bar{D}}{k} \\ &= \frac{18.83 \text{ cm}}{5880} \\ &= 0.0032 \text{ cm.}\end{aligned}$$

(6)

(b) Standard deviation is given by:

$$\sigma = \left[\frac{\sum_{i=1}^{10} f_i \delta_i^2}{N} \right] \quad (1.7)$$

The standard deviation σ_D of D is

$$\begin{aligned}\sigma_D &= \left[\frac{\sum_{i=1}^{10} f_i (D_i - \bar{D})^2}{\sum_{i=1}^{10} f_i} \right]^{\frac{1}{2}} \\ &= \left[\frac{438.22 \text{ cm}^2}{200} \right]^{\frac{1}{2}} \\ &= 1.48 \text{ cm.}\end{aligned}$$

The standard deviation σ_d from the mean diameter is therefore

$$\begin{aligned}\sigma_d &= \frac{\sigma_D}{k} \\ &= \frac{1.48 \text{ cm}}{5880} \\ &= 0.0003 \text{ cm.}\end{aligned}$$

(6)

(c) The intervals for the histogram in Figure 2 were chosen with D as the mid-value.

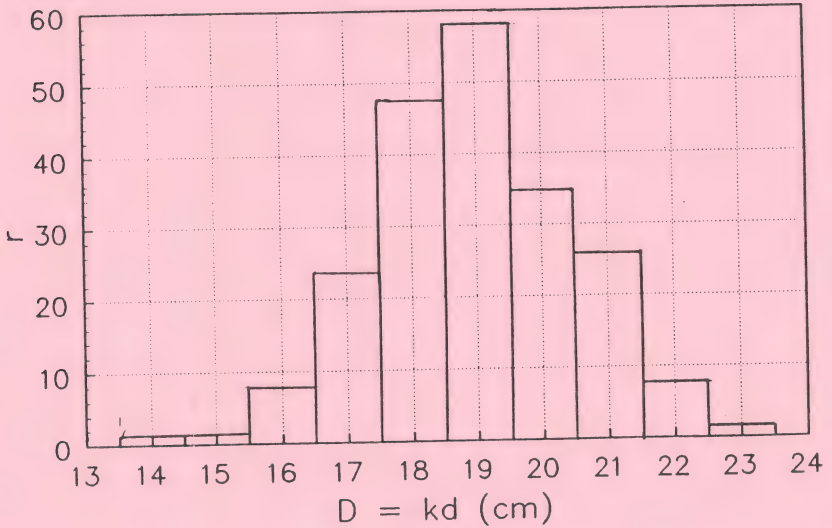


Figure 2: Histogram

(4)

(d) The equation for the normal frequency curve is

$$\begin{aligned}r &= \frac{1}{\sigma_D \sqrt{2\pi}} e^{\left[\frac{-(D-D)^2}{2\sigma_D^2} \right]} \\ &= \frac{1}{(1.48)\sqrt{2\pi}} e^{\left[\frac{-(D-18.83)^2}{2(1.48)^2} \right]} \\ &= 0.27 e^{-0.23(D-18.83)^2}\end{aligned}$$

The values of the relative frequency calculated from this equation are tabulated below:

kd	14	15	16	17	18	19	20	21	22	23
r	0.001	0.009	0.043	0.125	0.230	0.268	0.197	0.091	0.027	0.005

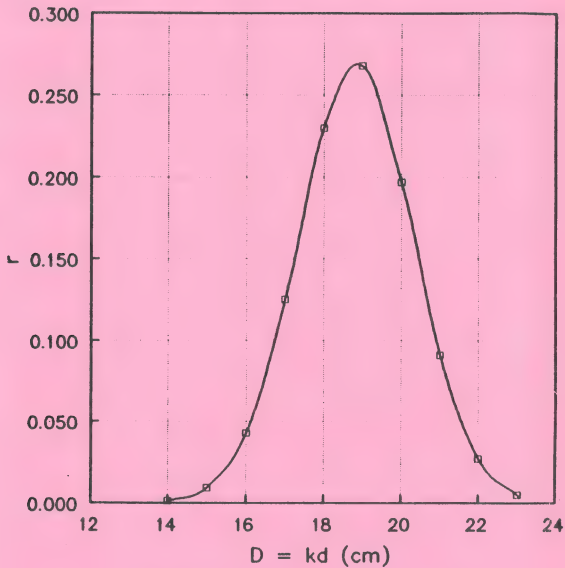


Figure 3: Normal frequency curve

(7)

[23]

2.1

It is given that

$$v = (40 \pm 2) \text{ km h}^{-1}$$

$$t = (4 \pm 0.25) \text{ h}$$

The distance travelled is

$$s = vt$$

$$\begin{aligned}
 &= (40 \text{ km h}^{-1})(4 \text{ h}) \\
 &= 160 \text{ km}
 \end{aligned}$$

The error on the distance travelled is calculated from the combination of the two errors, which is given by eq. (2.15). In this case it becomes

$$\frac{\Delta s}{s} = \left[\left(\frac{\Delta v}{v} \right)^2 + \left(\frac{\Delta y}{y} \right)^2 \right]^{\frac{1}{2}}$$

so that

$$\begin{aligned}
 \Delta s &= s \left[\left(\frac{\Delta v}{v} \right)^2 + \left(\frac{\Delta t}{t} \right)^2 \right]^{\frac{1}{2}} \\
 &= (160 \text{ km}) \left[\left(\frac{2}{40} \right)^2 + \left(\frac{0.25}{4} \right)^2 \right]^{\frac{1}{2}} \\
 &= 12.8 \text{ km}
 \end{aligned}$$

Thus

$$s = (160 \pm 13) \text{ km.}$$

[5]

2.6

Because D was measured 0.01 cm too large, its correct value is

$$\begin{aligned}
 D &= (3.04 - 0.01) \text{ cm} \\
 &= 0.03 \text{ cm}
 \end{aligned}$$

The correct value for V is therefore

$$\begin{aligned}
 V &= \frac{\pi D^3}{6} \\
 &= \frac{\pi (3.03 \text{ cm})^3}{6} \\
 &= 14.57 \text{ cm}^3
 \end{aligned}$$

The error on V follows from the rule for a scale factor (eq. (2.10)) and the rule for exponents (eq. (2.19)) so

$$\frac{\Delta V}{V} = a n \frac{\Delta D}{D}$$

where $a = \frac{\pi}{6}$ and $n = 3$.

Thus

$$\begin{aligned}\Delta V &= V \left[a n \frac{\Delta D}{D} \right] \\ &= (14.57 \text{ cm}^3) \left[\left(\frac{\pi}{6} \right) (3) \left(\frac{0.01}{3.03} \right) \right] \\ &= 0.08 \text{ cm}^3\end{aligned}$$

so finally

$$V = (14.57 \pm 0.08) \text{ cm}^3$$

[8]

2.10

Note that, in the case of addition and subtraction, it helps to write the numbers to the same exponent to determine the number of significant digits.

$$\begin{aligned}\text{(a)} \quad & (3.28 \times 10^5) + (4.25 \times 10^7) \\ &= (0.0328 \times 10^7) + (4.25 \times 10^7) \\ &= 4.2828 \times 10^7 \\ &\simeq 4.28 \times 10^7\end{aligned}$$

$$\begin{aligned}\text{(b)} \quad & (3.7 \times 10^6) + (2.91 \times 10^7) \\ &= (0.37 \times 10^7) + (2.91 \times 10^7) \\ &= 3.28 \times 10^7\end{aligned}$$

$$\begin{aligned}\text{(c)} \quad & (1.91 \times 10^{-3}) - (1.7 \times 10^{-5}) \\ &= (1.91 \times 10^{-3}) - (0.017 \times 10^{-3}) \\ &= 1.893 \times 10^{-3} \\ &\simeq 1.89 \times 10^{-3}\end{aligned}$$

$$\begin{aligned}\text{(d)} \quad & (5.48 \times 10^{-2})(2.6 \times 10^{-3}) \\ &= 1.4248 \times 10^{-4} \\ &\simeq 1.4 \times 10^{-4}\end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad & (6.77 \times 10^4)/(8.2347 \times 10^6) \\
 & = 8.2213 \times 10^{-3} \\
 & \simeq 8.22 \times 10^{-3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad & (2.1 \times 10^{-3})^2/(4.56 \times 10^{-2}) \\
 & = 9.671 \times 10^{-5} \\
 & \simeq 9.7 \times 10^{-5}
 \end{aligned}$$

[6]

3.1

To be able to plot a straight line graph, the equation

$$T = 2\pi \left[\frac{H - h}{g} \right]^{\frac{1}{2}} \quad (1)$$

has to be rewritten in the form $y = mx + c$. Note that the time for 50 oscillations was measured as a function of h , which makes T the dependent variable (to be plotted on the y-axis) and h the independent variable (to be plotted on the x-axis). From eq. (1)

$$\begin{aligned}
 T^2 &= 4\pi^2 \left[\frac{H - h}{g} \right] \\
 &= \frac{4\pi^2}{g} H - \frac{4\pi^2}{g} h \\
 &= -\frac{4\pi^2}{g} h + \frac{4\pi^2}{g} H \quad (2)
 \end{aligned}$$

Eq. (2) is now of the form

$$y = mx + c$$

with

$$\begin{aligned}
 y &= T^2 \\
 x &= h \\
 m &= -\frac{4\pi^2}{g} \\
 c &= \frac{4\pi^2}{g} H.
 \end{aligned}$$

To plot the graph of T^2 versus h , the following values were used (t refers to the time for 50 oscillations):

$h(\text{m})$	$t(\text{s})$	$T(\text{s})$	$T^2(\text{s}^2)$
0.4	155.3	3.106	9.647
0.6	148.8	2.976	8.866
0.8	142.2	2.844	8.088
1.0	134.0	2.680	7.182
1.2	127.4	2.548	6.492
1.4	119.2	2.384	5.684
1.6	110.5	2.210	4.884

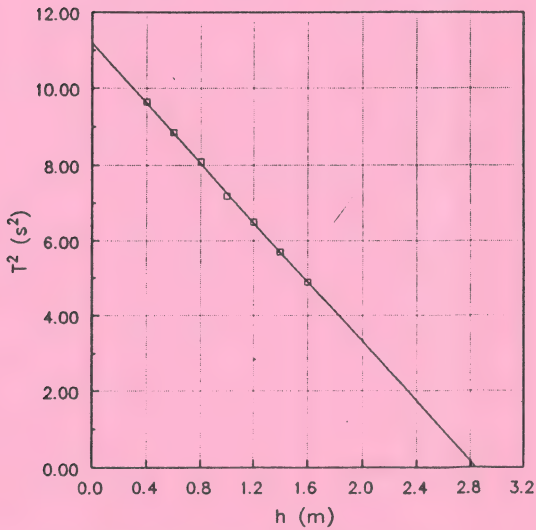


Figure 4: Graph of T^2 versus h

From the graph, the intercept on the ordinate is $c = 11.2 \text{ s}^2$, and from eq. (2)

$$c = \frac{4\pi^2 H}{g}$$

so

$$\begin{aligned}
 H &= \frac{c g}{4\pi^2} \\
 &= \frac{(11.2 \text{ s}^2)(9.8 \text{ m s}^{-2})}{4\pi^2} \\
 &= 2.78 \text{ m} \\
 &\simeq 2.8 \text{ m}
 \end{aligned}$$

From the graph, the intercept on the abscissa is $h = 2.83 \text{ m}$. From equation (2) it is clear that, because $T^2 = 0$ along the abscissa

$$\begin{aligned}
 \frac{4\pi^2}{g} h &= \frac{4\pi^2}{g} H \\
 \text{or } h &= H \\
 \text{so } H &= 2.83 \text{ m} \\
 &\simeq 2.8 \text{ m}
 \end{aligned}$$

(10)

Questions:

- (a) The value of H which was calculated using the intercept on the abscissa ($h = 2.83 \text{ m}$) is probably the least accurate, because it is situated furthest from the experimental points (the extrapolation to the abscissa is the greatest). A small error in the slope of the straight line can thus cause a big difference in the value of this intercept.

The accuracy can usually be improved by taking more readings. In this case there should also be an improvement if bigger values of h could be included in the results. This would decrease the distance between the rightmost datapoint and the intercept on the abscissa.

- (b) The formula $T = 2\pi\sqrt{\ell/g}$ only holds for small angles of oscillation. (Read section 8.4 (p. 177) in the study guide to see the derivation of this formula in which the approximation for small angles of oscillation is made).
- (c) The force of gravity on the bob acts through its centre of gravity, so the length of the pendulum is determined by the position of the centre of gravity (which is at the centre of the bob if we assume that it is of uniform density).

(d) The measuring errors at the beginning and end of a time interval are more or less constant, but the ratio of these errors to the total time interval becomes smaller when the time interval becomes larger.

(e) It is not necessary to use a ladder! (10)

[20]

3.10 Cauchy's equation

$$\begin{aligned}\mu &= A + B \frac{1}{\lambda^2} \\ &= B \frac{1}{\lambda^2} + A\end{aligned}$$

is of the form $y = mx + c$

with $y \equiv \mu$

$$x \equiv \frac{1}{\lambda^2}$$

$$m = B$$

$$c = A.$$

The values of A and B can be determined by means of linear regression. The summations needed for the calculation are tabulated below:

$\lambda(\text{\AA})$	$\frac{1}{\lambda^2}(\times 10^{-8} \text{\AA}^{-2})$	μ	$\frac{1}{\lambda^4}(\times 10^{-15} \text{\AA}^{-4})$	$\frac{\mu}{\lambda^2}(\times 10^{-8} \text{\AA}^{-2})$
2144	21.755	1.4040	47.328	30.544
3968	6.351	1.3435	4.034	8.533
4861	4.232	1.3371	1.791	5.659
5893	2.880	1.3330	0.829	3.839
6708	2.222	1.3307	0.494	2.957
7685	1.693	1.3289	0.287	2.250
Σ	17.378	6.6732	7.435	23.238

The graph of μ versus $1/\lambda^2$ shows that the first set of data in the table above is inaccurate and should therefore not be taken into account because Cauchy's equation will not be reliable for values of $\mu > 1.35$.

$$\begin{aligned}
 A = c &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2} \\
 &= \frac{\Sigma(1/\lambda^4) \Sigma \mu - \Sigma(1/\lambda^2) \Sigma(\mu/\lambda^2)}{n \Sigma(1/\lambda^4) - [\Sigma(1/\lambda^2)]^2} \\
 &= \frac{(7.435 \times 10^{-5} \text{Å}^{-4})(6.673) - (17.378 \times 10^{-8} \text{Å}^{-2})(23.238 \times 10^{-8} \text{Å}^{-2})}{(5)(7.435 \times 10^{-15} \text{Å}^{-4}) - (17.378 \times 10^{-8} \text{Å}^{-2})^2} \\
 &= 1.324
 \end{aligned}$$

$$\begin{aligned}
 B = m &= \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2} \\
 &= \frac{n \Sigma(\mu/\lambda^2) - \Sigma(1/\lambda^2) \Sigma \mu}{n \Sigma(1/\lambda^4) - [\Sigma(1/\lambda^2)]^2} \\
 &= \frac{(5)(23.238 \times 10^{-8} \text{Å}^{-2}) - (17.378 \times 10^{-8} \text{Å}^{-2})(6.673)}{(5)(7.435 \times 10^{-15} \text{Å}^{-4}) - (17.378 \times 10^{-8} \text{Å}^{-2})^2} \\
 &= 3.199 \times 10^5 \text{Å}^{-2}.
 \end{aligned}$$

To calculate a reference point through which the best straight line can be drawn, $1/\lambda^2 = 10 \times 10^{-8} \text{Å}^{-2}$ was chosen. For this value, the Cauchy equation renders $\mu = 1.356$ with the calculated values of A and B. The point (10^{-7} , 1.356) was marked as P on the graph, and the best straight line was drawn through P and

(0, 1.324).

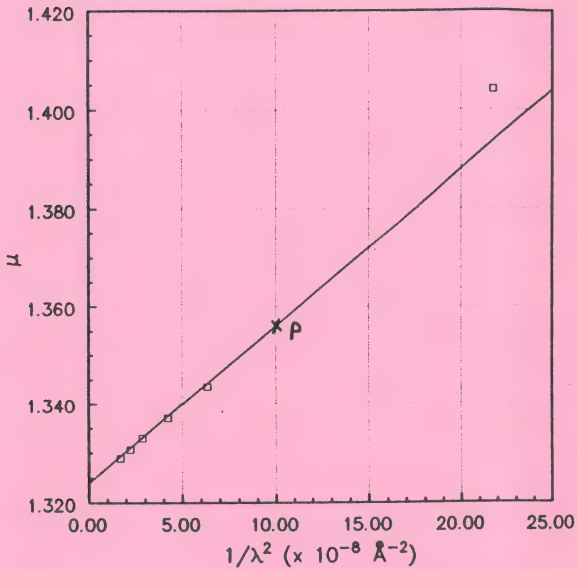
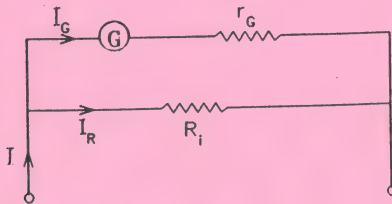


Figure 5: Graph of μ versus $1/\lambda^2$

[15]

5.4

If it is assumed that only one resistor is connected to the circuit at a time, the circuit can be presented as follows:



The current through the shunt resistor is

$$I_R = I - I_G$$

The potential difference across R is equal to the potential difference across the galvanometer, i.e.

$$V_R = I_R R = I_G r_G$$

and thus

$$\begin{aligned} R &= \frac{I_G r_G}{I_R} \\ &= \frac{I_G r_G}{I - I_G} \\ &= \frac{(0.01 \text{ A})(25 \Omega)}{I - 0.01 \text{ A}} \\ &= \frac{0.25 \text{ V}}{I - 0.01 \text{ A}} \end{aligned}$$

The value of R can now be calculated for each value of I :

i) For $I = 10 \text{ A}$;

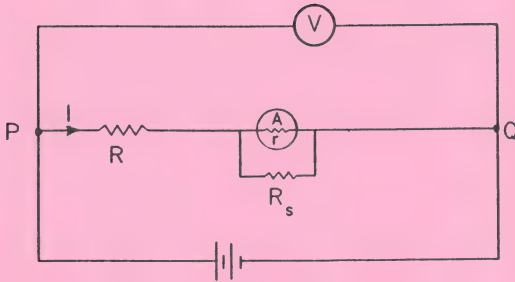
$$\begin{aligned} R_1 &= \frac{0.25 \text{ V}}{(10 - 0.01) \text{ A}} \\ &= 0.025 \Omega \end{aligned}$$

ii) For $I = 1 \text{ A}$;

$$\begin{aligned} R_2 &= \frac{0.25 \text{ V}}{(1 - 0.01) \text{ A}} \\ &= 0.253 \Omega \end{aligned}$$

iii) For $I = 0.1 \text{ A}$;

$$\begin{aligned} R_3 &= \frac{0.25 \text{ V}}{(0.1 - 0.01) \text{ A}} \\ &= 2.78 \Omega \end{aligned}$$

5.8

The effective resistance R_{eff} of the internal resistance r and the shunt resistance R_s is given by

$$\begin{aligned}\frac{1}{R_{eff}} &= \frac{1}{r} + \frac{1}{R_s} \\ &= \frac{R_s + r}{r R_s} \\ \therefore R_{eff} &= \frac{PrR_s}{R_s + r}\end{aligned}$$

The total resistance R_{tot} of the circuit is

$$\begin{aligned}R_{tot} &= R + R_{eff} \\ &= R + \frac{rR_s}{r + R_s}\end{aligned}$$

The potential difference V between points P and Q is given by:

$$\begin{aligned}V &= IR_{tot} \\ &= I \left(R + \frac{rR_s}{r + R_s} \right)\end{aligned}$$

Solving for R , we have

$$\begin{aligned}\frac{V}{I} &= R + \frac{rR_s}{r + R_s} \\ \Rightarrow R &= \frac{V}{I} - \frac{rR_s}{r + R_s} \\ &= \frac{V}{I} - \frac{R_s}{1 + R_s/r}\end{aligned}$$

If $R_s \ll r$, then $R_s/r \ll 1$ and

$$R = \frac{V}{I} - R_s.$$

[10]

MRS J A CILLIERS

Tel.: 429-8018

UNISA



UNISA UNIVERSITY OF SOUTH AFRICA



96194375

DEPARTMENT OF PHYSICS

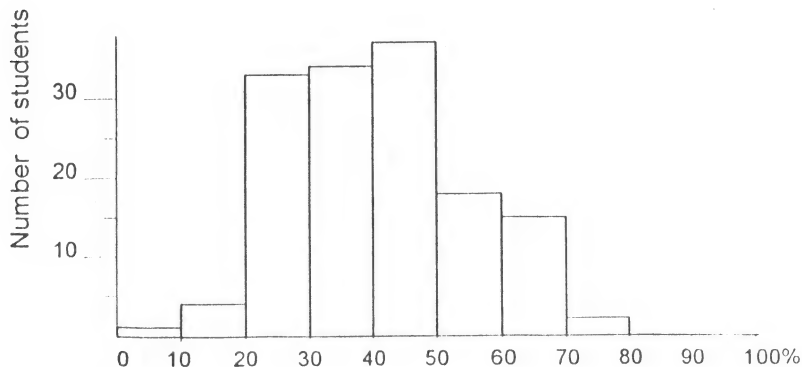
PHYSICS PHY103-8

TUTORIAL LETTER 104 – 1995

POST MORTEM FOR ASSIGNMENT 1

Dear Student,

As real life example, I have included a histogram of the marks obtained by students for this assignment.



The marks for the first assignment were rather low, but I am aware of the fact that because this was the first assignment of the year, it was not always exactly clear what was expected of you. Hang in there! It will get better from here.

Although the solutions in the key are written out in detail, there are some of the problems that warrant additional comments:

1.8

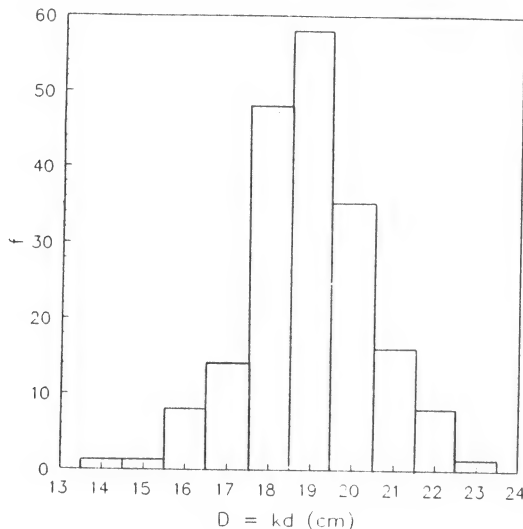
You will see that the graph in the key shows a curve, whereas the examples of frequency distributions in the study guide consist of vertical lines. This is an alternative presentation of the data, and full credit was given for either presentation as the question did not specify a particular presentation.

1.14

Unfortunately this solution contains two printing errors. In part (b) the formula for standard deviation should read

$$\sigma = \left[\frac{\sum_{i=1}^{10} f_i \delta_i^2}{N} \right]^{\frac{1}{2}} \quad (1.7)$$

The variable on the y-axis of the graph in part (c) is f and not r as indicated in Figure 2. Also, the frequency for $D = 21$ cm is 16 and not 26 as indicated on the histogram. The histogram should look like this:



Plotting the normal frequency curve in part (d) proved problematic. Few students realised that they should use eq. (1.9) to calculate the values of r . The solution in the key shows how you should do this calculation. Study it carefully and try to do the calculations yourself if you did not do so in your assignment.

2.1

This was a straightforward problem which most students answered correctly.

2.6

The solution for this problem in the key, is incorrect. It should read as follows:

$$\begin{aligned} D &= (3.04 - 0.01) \text{ cm} \\ &= 3.03 \text{ cm} \end{aligned}$$

The correct value of V is therefore

$$\begin{aligned} V &= \frac{\pi D^3}{6} \\ &= \frac{\pi (3.03 \text{ cm})^3}{6} \\ &= 14.57 \text{ cm}^3 \end{aligned}$$

The error in V follows from the rule for exponents, eq. (2.19), so

$$\frac{\Delta V}{V} = n \frac{\Delta D}{D}$$

where $n = 3$.

Thus

$$\begin{aligned} \Delta V &= V \left[n \frac{\Delta D}{D} \right] \\ &= (14.57 \text{ cm}^3) \left[(3) \left(\frac{0.01 \text{ cm}}{3.03 \text{ cm}} \right) \right] \\ &= 0.14 \text{ cm}^3 \end{aligned}$$

so finally

$$V = (14.57 \pm 0.14) \text{ cm}^3.$$

For those of you who understand calculus, the following explains why the scale factor is not included when the rule for exponents is applied:

When

$$z = cx^a$$

we have

$$\begin{aligned}\frac{dz}{dx} &= c a x^{a-1} \\ &= a \frac{cx^a}{x} \\ &= a \frac{z}{x}\end{aligned}$$

so that

$$\frac{dz}{z} = a \frac{dx}{x}.$$

When, however,

$$z = cx$$

we have

$$\frac{dz}{dx} = c$$

so that

$$dz = c dx$$

2.10

From the answers to this question, it is apparent that significant numbers presents a problem to most students. STUDY SECTION 2.4 ON SIGNIFICANT FIGURES IN THE STUDY GUIDE AGAIN, CAREFULLY.

Apparently the rule for adding and subtracting gives the most trouble. The best advice I can give in this regard, is to write the numbers you are adding or subtracting to the same exponent, as indicated in the key. Remember: *The answer contains as many places after the decimal point, as the least accurate number in the sum.*

3.1

This was a rather difficult problem, so don't be too upset if you did not do well in it!

The solution in the key is written out in detail, and shows you exactly what was expected. The one point which most students missed, is that one can determine two values for H from the graph - one from the x-intercept, and one from the y-intercept. Question (a) refers to these two values of H , and not to one of the data points!

Many students plotted the total time t for 50 oscillations instead of the period. Take note that the T in the table on p. 9 of the key is the PERIOD of the oscillation, i.e. the time for ONE oscillation, $T = t/50$.

3.10

From the way in which this question was formulated, it was not clear that a graph was required, but doing all the calculations for the linear regression really implies that one wants to use them to plot a graph.

Please note that the summations given in the key, exclude the first datapoint, because it is inaccurate.

5.4

This problem was generally well answered.

5.8

I am aware that the formulation of this problem was not clear. The galvanometer which it refers to, is by implication a galvanometer being used as an ammeter by placing a shunt resistance across it, as explained in section 5.4.2 in the study guide.

The solution in the key shows you what was meant and how you can prove the given relation. Unfortunately, the fifth line of the solution contains a printing error. It should read

$$\therefore R_{eff} = \frac{rR_s}{R_s + r}$$

I hope that these comments have helped you in the areas where you experience difficulty. All the best with your studies!

MRS J A CILLIERS

Tel.: 429-8018

UNISA



UNISA UNIVERSITY OF SOUTH AFRICA



96209615

DEPARTMENT OF PHYSICS

PHYSICS PHY103-8

TUTORIAL LETTER 202 – 1995

KEY FOR ASSIGNMENT 2

6.8

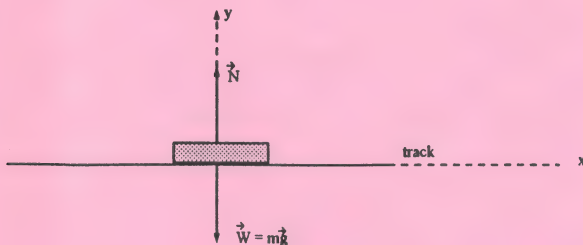


Figure 1: Forces acting on the glider

The forces acting on the glider while it is moving at constant velocity on the flat, horizontal air track, are shown in Fig. 1. The force $\vec{W}$ is the weight of the glider. As you know, the weight of the glider is the force $m\vec{g}$ exerted on the glider by

the earth, and it is always directed vertically downwards. $\vec{N}$ is the upward force exerted on the glider by the air. This force is always perpendicular to the air track. Because the air track is exactly horizontal, the angle between $\vec{W}$ and $\vec{N}$ is 180° and resultant force is given by

$$\vec{R} = \vec{N} + \vec{W}$$

$\Sigma F_y = 0$ because the glider does not move in the vertical direction, i.e.

$$\Sigma F_y = \vec{R} = 0.$$

The resultant force $\vec{R}$ has no component in the x-direction, so

$$\Sigma F_x = 0$$

and therefore the glider will move at a constant velocity according to Newton's first law. [5]

6.14

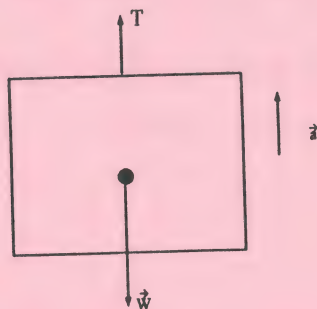


Figure 2: Forces acting on the elevator

The upward acceleration which brings the elevator to rest, follows from

$$v^2 = u^2 + 2as$$

i.e.

$$\begin{aligned} a &= \frac{v^2 - u^2}{2s} \\ &= \frac{0 - (10 \text{ m s}^{-1})^2}{2(15 \text{ m})} \\ &= -3.33 \text{ m s}^{-2} \end{aligned}$$

The negative sign indicates that the acceleration is in the opposite direction to the motion of the elevator.

From Newton's second law, it follows that

$$\Sigma F_y = ma$$

or

$$T - W = ma$$

where $m = W/g$ so

$$T - W = \frac{W}{g} a$$

and

$$\begin{aligned} T &= W + \frac{W}{g} a \\ &= W \left(1 + \frac{a}{g} \right) \\ &= (12.2 \times 10^3 \text{ N}) \left(1 + \frac{3.33 \text{ m s}^{-2}}{9.8 \text{ m s}^{-2}} \right) \\ &= 1.6 \times 10^4 \text{ N.} \end{aligned}$$

[10]

6.19

The angle θ follows from

$$\begin{aligned} \sin \theta &= \frac{\text{Thickness of block}}{\text{Distance between setting screws}} \\ &= \frac{0.02 \text{ m}}{2.5 \text{ m}} \end{aligned}$$

or

$$\begin{aligned} \theta &= \sin^{-1} \left(\frac{0.02 \text{ m}}{2.5 \text{ m}} \right) \\ &= 0.46^\circ \end{aligned}$$

The initial velocity of the glider v_{0x} is given by

$$\begin{aligned} v_{0x} &= \frac{\text{Width of flag}}{t_A} \\ &= \frac{2 \text{ cm}}{t_A} \end{aligned}$$

and the final velocity is given by

$$\begin{aligned} v_{0x} &= \frac{\text{Width of flag}}{t_C} \\ &= \frac{2 \text{ cm}}{t_C} \end{aligned}$$

The time t_B recorded on timer T_B gives the time t it takes the glider to cover the distance x .

The values calculated for v_{0x} and v_x are tabulated below:

x (m)	v_{0x} (10^{-2} m s^{-2})	v_x (m s^{-1})	t (s)
0.1	4.85	1.25	15.400
0.5	4.90	2.60	36.010
1.0	4.95	3.39	50.020
1.5	4.83	4.08	61.010
1.9	5.00	6.67	69.341
2.1	4.74	6.06	74.009
2.5	4.87	5.26	78.099

To plot v_x as a function of t , we use eq. (6.9)

$$\begin{aligned} v_x &= v_{0x} + gt \sin \theta \\ &= (g \sin \theta)t + v_{0x} \end{aligned}$$

This equation is of the form

$$y = mx + c$$

with $y \equiv v_x$, $x \equiv t$, $m \equiv g \sin \theta$, $c \equiv v_{0x}$.

The summations needed for the linear regression analysis are tabulated below:

i	x_i (s)	y_i (m s ⁻¹)	$x_i y_i$ (10 ² m)	x_i^2 (10 ³ s ²)	$m x_i + c$ (m s ⁻¹)	δ_i^2 (m ² s ⁻²)
1	15.400	1.25	0.193	0.237	1.05	0.040
2	36.010	2.60	0.936	1.297	2.69	0.008
3	50.020	3.39	1.696	2.502	3.81	0.176
4	61.010	4.08	2.489	3.722	4.69	0.372
5	69.341	6.67	4.625	4.808	5.35	1.742
6	74.009	6.06	4.485	5.477	5.72	0.116
7	78.099	5.26	4.108	6.099	6.05	0.624
$\sum_{i=1}^7$	383.889	29.31	18.532	24.142		3.078

$$\begin{aligned}
 D &= n \Sigma x^2 - [\Sigma x]^2 \\
 &= (7)(24.142 \times 10^3 \text{ s}^2) - (383.889 \text{ s})^2 \\
 &= 2.16 \times 10^4 \text{ s}^2
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m &= \frac{n \Sigma xy - \Sigma x \Sigma y}{D} \\
 &= \frac{7(18.532 \times 10^2 \text{ m}) - (383.889 \text{ s})(29.31 \text{ m s}^{-1})}{2.16 \times 10^4 \text{ s}^2} \\
 &= 7.97 \times 10^{-2} \text{ m s}^{-2}
 \end{aligned}$$

and the y-intercept is

$$\begin{aligned}
 c &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{D} \\
 &= \frac{(24.142 \times 10^3 \text{ s}^2)(29.31 \text{ m s}^{-1}) - (383.889 \text{ s})(18.532 \times 10^2 \text{ m})}{2.16 \times 10^4 \text{ s}^2} \\
 &= -0.177 \text{ m s}^{-1}
 \end{aligned}$$

The errors in m and c are given by

$$\begin{aligned}
 s_m &= \left[\left(\frac{n}{n-2} \right) \frac{\Sigma \delta_i^2}{D} \right]^{\frac{1}{2}} \\
 &= \left[\left(\frac{7}{5} \right) \left(\frac{3.078 \text{ m}^2 \text{ s}^{-2}}{2.16 \times 10^4 \text{ s}^2} \right) \right]^{\frac{1}{2}}
 \end{aligned}$$

$$= 0.014 \text{ m s}^{-2}$$

$$\begin{aligned} s_c &= \left[\left(\frac{1}{n-2} \right) \frac{\Sigma \delta_i^2 \Sigma x^2}{D} \right]^{\frac{1}{2}} \\ &= \left[\left(\frac{1}{5} \right) \frac{(3.078 \text{ m}^2 \text{ s}^{-2})(24.142 \times 10^3 \text{ s}^2)}{2.16 \times 10^4 \text{ s}^2} \right]^{\frac{1}{2}} \\ &= 0.829 \text{ m s}^{-1} \end{aligned}$$

The value of g follows from the slope,

$$\begin{aligned} m &= g \sin \theta \\ \Rightarrow g &= \frac{m}{\sin \theta} \\ &= \frac{7.97 \times 10^{-2} \text{ m s}^{-2}}{8 \times 10^{-3}} \\ &= 9.96 \text{ m s}^{-2} \end{aligned}$$

The error on g follows from eq. (2.10) as

$$\begin{aligned} \Delta g &= \left(\frac{1}{\sin \theta} \right) \Delta m \\ &= \frac{(0.014 \text{ m s}^{-2})}{8 \times 10^{-3}} \\ &= 1.75 \text{ m s}^{-2} \end{aligned}$$

$$\text{Thus } g \simeq (10 \pm 2) \text{ m s}^{-2}.$$

From the y-intercept

$$v_0 x = (-0.18 \pm 0.82) \text{ m s}^{-1}$$

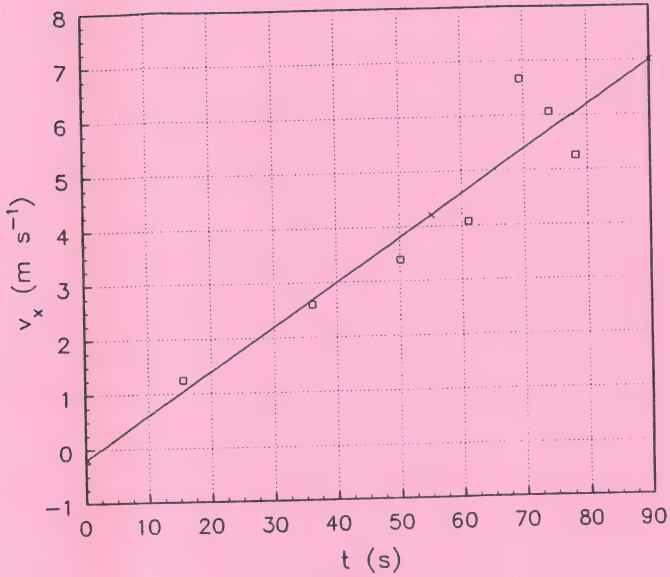


Figure 3: Graph of v_x versus t

To plot a graph of v_x^2 versus x , we use eq. (6.11)

$$\begin{aligned} v_x^2 &= v_{0x}^2 + 2g \sin \theta x \\ &= (2g \sin \theta)x + v_{0x}^2 \end{aligned}$$

which is of the form

$$y = mx + c$$

with

$$y \equiv v_x^2, \quad x \equiv x, \quad m \equiv 2g \sin \theta, \quad c \equiv v_{0x}^2$$

The summations needed for the linear regression analysis are tabulated below:

i	x_i (m)	y_i (m ² s ⁻²)	$x_i y_i$ (m ³ s ⁻²)	x_i^2 (m ²)	$m x_i + c$ (m ² s ⁻²)	δ_i^2 (m ⁴ s ⁻⁴)
1	0.1	1.56	0.156	0.01	1.06	0.25
2	0.5	6.76	3.38	0.25	7.26	0.25
3	1.0	11.49	11.49	1.00	15.01	1.24
4	1.5	16.65	24.98	2.25	22.76	37.33(!)
5	1.9	44.49	84.53	3.61	28.96	241.18(!)
6	2.1	36.72	77.11	4.41	32.06	21.71(!)
7	2.5	27.67	69.18	6.25	38.26	112.15(!)
$\sum_{i=1}^7$	9.6	145.34	270.83	17.78		414.11(!)

From the large values of δ_i^2 it is clear that there is a snake in the grass, but let us continue with the calculations to see where they lead.

$$\begin{aligned}
 D &= n\Sigma x^2 - [\Sigma x]^2 \\
 &= (7)(17.78 \text{ s}^2) - (9.6 \text{ m})^2 \\
 &= 32.3 \text{ m}^2
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m &= \frac{n\Sigma xy - \Sigma x \Sigma y}{D} \\
 &= \frac{(7)(270.83 \text{ m}^3 \text{ s}^{-2}) - (9.6 \text{ m})(145.34 \text{ m}^2 \text{ s}^{-2})}{32.3 \text{ m}^2} \\
 &= 15.50 \text{ m s}^{-2}
 \end{aligned}$$

and the y-intercept is

$$\begin{aligned}
 c &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{D} \\
 &= \frac{(17.78 \text{ m}^2)(145.34 \text{ m}^2 \text{ s}^{-2}) - (9.6 \text{ m})(270.83 \text{ m}^3 \text{ s}^{-2})}{32.3 \text{ m}^2} \\
 &= -0.49 \text{ m}^2 \text{ s}^{-2}
 \end{aligned}$$

The errors in m and c are given by

$$s_m = \left[\left(\frac{n}{n-2} \right) \left(\frac{\Sigma \delta_i^2}{D} \right) \right]^{\frac{1}{2}}$$

$$\begin{aligned}
 &= \left[\left(\frac{7}{5} \right) \left(\frac{414.11 \text{ m}^4 \text{ s}^{-4}}{32.3 \text{ m}^2} \right) \right]^{\frac{1}{2}} \\
 &= 4.24 \text{ m s}^{-2} \\
 \text{and } s_c &= \left[\left(\frac{1}{n-2} \right) \left(\frac{\Sigma \delta_i^2 \Sigma x^2}{D} \right) \right]^{\frac{1}{2}} \\
 &= \left[\left(\frac{1}{5} \right) \frac{(414.11 \text{ m}^4 \text{ s}^{-4})(17.78 \text{ m}^2)}{32.3 \text{ m}^2} \right]^{\frac{1}{2}} \\
 &= 6.75 \text{ m}^2 \text{ s}^{-2}
 \end{aligned}$$

Thus

$$\begin{aligned}
 g &= \frac{m}{2 \sin \theta} \\
 &= \frac{15.50 \text{ m s}^{-2}}{2(8 \times 10^{-3})} \\
 &= 969 \text{ m s}^{-2}(!!)
 \end{aligned}$$

and the error on g is

$$\begin{aligned}
 \Delta g &= \left(\frac{1}{2 \sin \theta} \right) \Delta m \\
 &= \left(\frac{1}{1.6 \times 10^{-2}} \right) (4.24 \text{ m s}^{-2}) \\
 &= 265 \text{ m s}^{-2} \\
 \therefore g &= (1000 \pm 270) \text{ m s}^{-2}(!)
 \end{aligned}$$

From this result it is clear that the data is extremely unreliable, and probably contains a printing error!

Graph of x versus t :

$$x = v_{0x}t + \frac{1}{2}g \sin \theta t^2.$$

We know that

$$v_x = \frac{dx}{dt}$$

so

$$v_{0x} = \left(\frac{dx}{dt} \right)_{t=0}$$

Without calculus

$$v_x = \frac{\Delta x}{\Delta t} = \text{slope}$$

$$v_{0x} = v_x \quad \text{when} \quad t = 0 \quad (\text{from } v_x = v_{0x} + at)$$

The magnitude of the velocity at any point on the graph is therefore given by the slope the tangent to of the graph at that point.

From the graph

$$\Delta x \approx 0 \quad \text{at} \quad t = 0$$

$$\Rightarrow v_{0x} \approx 0.$$

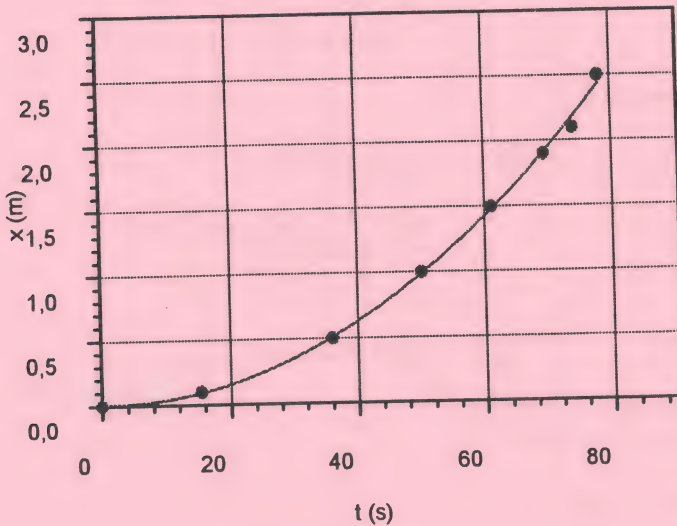


Figure 4: Graph of x versus t

[24]

7.5

Any object which moves in a circular path, is acted on by a centripetal force. An atom near the end of the aluminium bar therefore also experiences a centripetal

force. In this case the force is provided by the attracting forces between the aluminium atom and its neighbours. [2]

7.12

a) Experiment 2(a)

In this experiment the metal sphere experiences the following forces:

- i) When the apparatus is stationary, the forces acting on the metal are as indicated in Figure 5. $\vec{N}$ is the normal force exerted by the cylinder on the sphere, and $\vec{W} = m\vec{g}$ is the force exerted by the earth on the sphere.

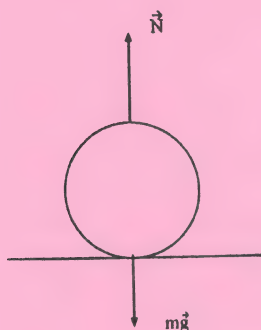


Figure 5: Force diagram when apparatus is at rest

There is no applied force and thus no motion in the horizontal direction, so the frictional force is also zero in this case. We have

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0 : \quad \vec{W} = \vec{N} = 0$$

- ii) When the apparatus is set in motion the forces acting on the sphere are those shown in Figure 6.

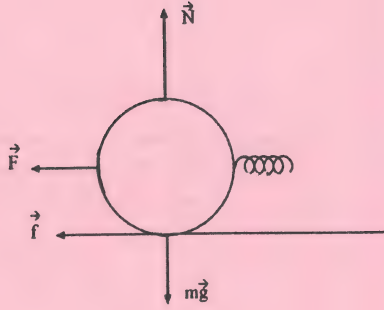


Figure 6: Force diagram when apparatus is rotating

Again, $\vec{N}$ is the force exerted by the cylinder on the sphere, and $\vec{W}$ is the force exerted by the earth on the sphere. $\vec{F}$ is the force exerted by the spring on the sphere and $\vec{f}$ is the resultant frictional force between the sphere and the cylinder.

Note that when the apparatus is set in motion, and the speed is gradually increased, the radius of the circular path followed by the sphere will also be increased. The motion of the sphere relative to the tube will therefore be directed outwards, and since the frictional force tends to oppose this motion, it is directed toward the centre of the circular path, i.e. in the SAME DIRECTION as $\vec{F}$.

This implies that when the frictional force is taken into account, the magnitude of the resultant force on the sphere is

$$\Sigma F_x = F + f$$

toward the centre of motion. Since the sphere executes uniform circular motion, ΣF_x represents the centripetal force and therefore

$$F + f = \frac{mv^2}{r}$$

According to Hooke's law, the magnitude of the force exerted by the spring on the sphere is $F = kx$, so

$$kx + f = \frac{mv^2}{r}$$

or

$$x + \frac{f}{k} = \left(\frac{m}{k}\right) \left(\frac{v^2}{r}\right)$$

from which

$$x = \left(\frac{m}{k}\right) \left(\frac{v^2}{r}\right) - \frac{f}{k}.$$

A graph of x versus v^2/r would yield a straight line with slope m/k and intercept $-f/k$ on the ordinate. If k is known, the magnitude of frictional force can be estimated from the value of this intercept.

In the experiment the frictional force is not taken into account, and the centripetal force is assumed to be the force $F = kx$, i.e.

$$kx = \frac{mv^2}{r}$$

and thus

$$x = \frac{m}{k} \frac{v^2}{r}$$

In this case the graph of x versus v^2/r would yield a straight line with slope m/k , passing through the origin.

b) Experiment 2(b)

In experiment 2(b) frictional forces are present between both the metal cylinders and the perspex tube. Let us consider the forces on the two masses m and M when the apparatus is rotating and M is in equilibrium without touching the floor of the tube. The forces experienced by masses

m and M are indicated in Figures 7(a) and 7(b) respectively.

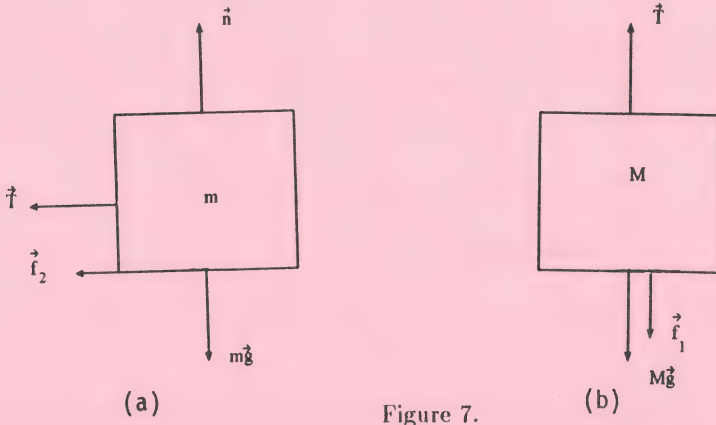


Figure 7.

$\vec{T}$ is the force exerted by the wire on both masses M and m , $M\vec{g}$ is the force exerted by the earth on mass M , $m\vec{g}$ is the force exerted by the earth on mass m and $\vec{n}$ is the force exerted by the tube on mass m .

Let the resultant frictional forces on M and m be $\vec{f}_1$ and $\vec{f}_2$ respectively. When the apparatus is put into motion, the speed of m gradually increases, and the radius of the circular path along which it is travelling also increases. The motion of m relative to the cylinder is therefore directed outwards and because the frictional force will tend to prevent this motion, $\vec{f}_2$ is directed towards the centre of the circle. When the speed of m is great enough, the mass M will start moving upwards and the frictional force $\vec{f}_1$ between M and the tube will therefore be directed downwards.

When M is in equilibrium

$$\Sigma F_y = 0$$

$$\therefore T - f_1 - Mg = 0$$

Thus

$$T = Mg + f_1 \quad (1)$$

and the resultant inward force exerted on m is $T + f_2$. Because m is executing circular motion

$$T + f_2 = \frac{mv^2}{R}$$

Substituting for T from eq. (1) renders

$$(Mg + f_1) + f_2 = \frac{mv^2}{R}$$

or

$$Mg + (f_1 + f_2) = \frac{mv^2}{R}$$

which accounts for a systematic error when determining M in the experiment. [10]

7.19

The motion of the sphere is described by eq. (7.6)

$$\frac{v^2}{R} = \left(\frac{k}{m} \right) x$$

which is of the form

$$y = m'x + c$$

with

$$y \equiv \frac{v^2}{R}, \quad x \equiv x, \quad m' \equiv k/m \quad \text{and} \quad c = 0.$$

The speed v is calculated from the relation

$$v = \frac{2\pi R}{T}$$

where T is the period of the motion. The time t for 30 revolutions is given, so

$$T = \frac{t}{30}.$$

x (m)	t (30 rev.) (s)	$R = t/30$ (s)	$R = a + x$ (m)	$v = 2\pi R/T$ (m s ⁻¹)	v^2/R (m s ⁻²)
0.02	154	5.13	0.12	0.147	0.18
0.04	116	3.87	0.14	0.227	0.37
0.07	98	3.27	0.17	0.327	0.63
0.12	84	2.80	0.22	0.494	1.11
0.18	77	2.57	0.28	0.685	1.68
0.23	76	2.53	0.33	0.820	2.04
0.25	74	2.47	0.35	0.890	2.26
0.32	72	2.40	0.42	1.100	2.88

The summations needed for the linear regression analysis are tabulated below:

i	x_i (m)	y_i (m s ⁻²)	$x_i y_i$ (10 ⁻¹ m ² s ⁻²)	x_i^2 (10 ⁻² m ²)	$m'x_i + c$ (m s ⁻²)	δ_i^2 (10 ⁻³ m ² s ⁻⁴)
1	0.02	0.18	0.036	0.04	0.193	0.169
2	0.04	0.37	0.148	0.16	0.373	0.009
3	0.07	0.63	0.441	0.49	0.642	0.144
4	0.12	1.11	1.332	1.44	1.091	0.361
5	0.18	1.68	3.024	3.24	1.629	2.601
6	0.23	2.04	4.692	5.29	2.078	1.444
7	0.25	2.26	5.650	6.25	2.258	0.004
8	0.32	2.88	9.216	10.24	2.886	0.036
$\sum_{i=1}^8$	1.23	11.15	24.539	27.15		4.768

$$\begin{aligned}
 D &= n\Sigma x^2 - (\Sigma x)^2 \\
 &= 8(27.15 \times 10^{-2} \text{ m}^2) - (1.23 \text{ m})^2 \\
 &= 0.6591 \text{ m}^2
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m' &= \frac{n\Sigma xy - \Sigma x \Sigma y}{D} \\
 &= \frac{(8)(24.539 \times 10^{-1} \text{ m}^2 \text{ s}^{-2}) - (1.23 \text{ m})(11.15 \text{ m s}^{-2})}{0.6591 \text{ m}^2} \\
 &= 8.977 \text{ s}^{-2}
 \end{aligned}$$

and the y-intercept is

$$\begin{aligned}
 c &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{D} \\
 &= \frac{(27.15 \times 10^{-2} \text{ m}^2)(11.15 \text{ m s}^{-2}) - (1.23)(24.539 \times 10^{-1} \text{ m}^2 \text{ s}^{-2})}{0.65913 \text{ m}^2} \\
 &= 1.355 \times 10^{-2} \text{ m s}^{-2}.
 \end{aligned}$$

The errors in m and c are given by

$$\begin{aligned} s_m &= \left[\left(\frac{n}{n-2} \right) \left(\frac{\Sigma \delta_i^2}{D} \right) \right]^{\frac{1}{2}} \\ &= \left[\frac{(8)(4.768 \times 10^{-3} \text{ m}^2 \text{ s}^{-4})}{(6)(0.6591 \text{ m}^2)} \right]^{\frac{1}{2}} \\ &= 9.82 \times 10^{-3} \text{ s}^{-2} \end{aligned}$$

and

$$\begin{aligned} s_c &= \left[\left(\frac{1}{n-2} \right) \left(\frac{\Sigma \delta_i^2 \Sigma x^2}{D} \right) \right]^{\frac{1}{2}} \\ &= \left[\frac{(4.768 \times 10^{-3} \text{ m}^2 \text{ s}^{-4})(27.15 \times 10^{-2} \text{ m}^2)}{6(0.6591 \text{ m}^2)} \right]^{\frac{1}{2}} \\ &= 1.81 \times 10^{-2} \text{ m s}^{-2} \end{aligned}$$

Thus

$$\frac{k}{m} = (8.98 \pm 0.10) \text{ s}^{-2}$$

and

$$c = (1.4 \pm 1.8) \times 10^{-2} \text{ m s}^{-2}.$$

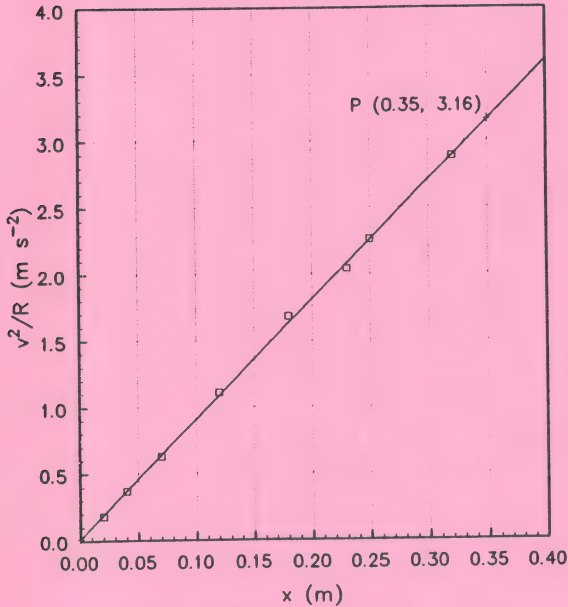


Figure 8: Graph of v^2/R versus x .

8.2

For two springs, each with spring constant k , connected in series, the effective spring constant k' is given by

$$\begin{aligned}\frac{1}{k'} &= \frac{1}{k} + \frac{1}{k} \\ \therefore k' &= \frac{k}{2}\end{aligned}\quad (1)$$

When a mass m attached to a spring is in equilibrium with the spring extended by x , the force kx exerted by the spring on the mass is equal to the weight $m\vec{g}$ of the mass, i.e.

$$\begin{aligned}\Sigma F_y = 0: \quad kx - mg &= 0 \\ \Rightarrow \quad kx &= mg\end{aligned}\quad (2)$$

When the same mass is attached to two springs in series, and it is in equilibrium with the springs extended by x' , i.e.

$$\begin{aligned}\Sigma F_y = 0: \quad k'x' - mg &= 0 \\ \text{or} \quad k'x' &= mg\end{aligned}$$

Substitution for k' renders

$$\frac{k}{2} x' = mg$$

and from (2)

$$\frac{k}{2} x' = kx.$$

Thus

$$x' = 2x$$

[6]

8.16

According to eq. (8.12) the period T of a simple pendulum of length ℓ is

$$\begin{aligned}T &= 2\pi\sqrt{\frac{\ell}{g}} \\ \text{or} \quad T^2 &= \frac{4\pi^2}{g} \ell \\ \text{so} \quad \ell &= \frac{g}{4\pi^2} T^2.\end{aligned}$$

For a second pendulum $T = 2$ s, so the length

$$\begin{aligned}\ell &= \frac{(9.8 \text{ m s}^{-2})}{4\pi^2} (2 \text{ s})^2 \\ &= 0.99 \text{ m}.\end{aligned}$$

[3]

8.23

The period T of a simple pendulum of length ℓ is given by eq. (8.12)

$$T = 2\pi \sqrt{\frac{\ell}{g}}$$

or

$$T^2 = \frac{4\pi^2}{g} \ell$$

which is of the form

$$y = mx + c$$

where

$$y \equiv T^2, \quad x \equiv \ell, \quad m \equiv \frac{4\pi^2}{g} \quad \text{and} \quad c = 0.$$

The average radius of the bob is

$$\begin{aligned}F &= \frac{\Sigma r_i}{n} \\ &= \frac{1.83 \text{ cm}}{6} \\ &= 0.31 \text{ cm}.\end{aligned}$$

The adjusted standard deviation is

$$\begin{aligned}s &= \left[\frac{\Sigma f_i \delta_i^2}{n-1} \right]^{\frac{1}{2}} \\ &= \left[\frac{2(0.01)^2 + (0.02)^2}{5} \right]^{\frac{1}{2}} \text{ cm} \\ &= 0.011 \text{ cm}.\end{aligned}$$

Thus

$$\begin{aligned}\bar{r} &= (0.31 \pm 0.01) \text{ cm} \\ &= (3.1 \pm 0.1) \times 10^{-3} \text{ m}.\end{aligned}$$

and the length ℓ of the pendulum is

$$\ell = \ell' + \bar{r}$$

where ℓ' is the length of the string.

The time t for 80 oscillations is given, so the period T is

$$T = \frac{t}{80}$$

ℓ' (m)	$\ell = \ell' + \bar{r}$ (m)	$t = (80 \text{ osc.})$ (s)	$T = t/80$ (s)	T^2 (s ²)
0.053	0.0561	45.2	0.5650	0.3192
0.108	0.1111	66.3	0.8288	0.6869
0.163	0.1661	83.2	1.0400	1.0816
0.283	0.2861	112.1	1.4012	1.9634
0.382	0.3851	122.4	1.5300	2.3409
0.550	0.5531	150.2	1.8775	3.5250

The summations needed for a linear regression analysis are tabulated below:

i	x_i (m)	y_i (s ²)	$x_i y_i$ (m s ²)	x_i^2 (m ²)	$m x_i + c$ (s ²)	δ_i^2 (10 ⁻² s ⁴)
1	0.056	0.319	0.0179	0.0031	0.3575	0.1482
2	0.111	0.687	0.0763	0.0123	0.7076	0.0424
3	0.166	1.082	0.1796	0.0276	1.0577	0.0590
4	0.286	1.963	0.5614	0.0818	1.8214	2.0051
5	0.385	2.341	0.9013	0.1482	2.4515	1.2210
6	0.553	3.525	1.9493	0.3058	3.5208	0.0018
$\sum_{i=1}^8$	1.557	9.917	3.6858	0.5788		3.4775

$$\begin{aligned}
 D &= n \Sigma x^2 - (\Sigma x)^2 \\
 &= (6)(0.5788 \text{ m}^2) - (1.557 \text{ m})^2 \\
 &= 1.0486 \text{ m}^2
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m &= \frac{n\Sigma xy - \Sigma x\Sigma y}{D} \\
 &= \frac{6(3.6858 \text{ m s}^2) - (1.557 \text{ m})(9.917 \text{ s}^2)}{1.0486 \text{ m}^2} \\
 &= 6.3647 \text{ s}^{-2} \text{ m}^{-1}
 \end{aligned}$$

and the y-intercept is

$$\begin{aligned}
 c &= \frac{\Sigma x^2\Sigma y - \Sigma x\Sigma xy}{D} \\
 &= \frac{(0.5788 \text{ m}^2)(9.917 \text{ s}^2) - (1.557 \text{ m})(3.6858 \text{ s}^2)}{1.0486 \text{ m}^2} \\
 &= 1.1148 \times 10^{-3} \text{ s}^2.
 \end{aligned}$$

The errors in m and c are given by

$$\begin{aligned}
 s_m &= \left[\left(\frac{n}{n-2} \right) \left(\frac{\Sigma \delta_i^2}{D} \right) \right]^{\frac{1}{2}} \\
 &= \left[\frac{(6)(3.4775 \times 10^{-2} \text{ s}^4)}{(4)(1.0486 \text{ m}^2)} \right]^{\frac{1}{2}} \\
 &= 2.23 \times 10^{-1} \text{ s}^2 \text{ m}^{-1}
 \end{aligned}$$

and

$$\begin{aligned}
 s_c &= \left[\left(\frac{1}{n-2} \right) \left(\frac{\Sigma \delta_i^2 \Sigma x^2}{D} \right) \right]^{\frac{1}{2}} \\
 &= \left[\frac{(3.4775 \times 10^{-2} \text{ s}^4)(0.5788 \text{ m}^2)}{(4)(1.0486)} \right]^{\frac{1}{2}} \\
 &= 6.93 \times 10^{-2} \text{ s}^2.
 \end{aligned}$$

Thus

$$m \simeq (6.36 \pm 0.22) \text{ s}^2 \text{ m}^{-1}$$

and

$$c \simeq (0.11 \pm 6.9) \times 10^{-2} \text{ s}^2.$$

The value of g follows from the slope:

$$m = \frac{4\pi^2}{g}$$

so

$$\begin{aligned}
 g &= \frac{4\pi^2}{m} \\
 &= \frac{4\pi^2}{6.36 \text{ s}^2 \text{ m}^{-1}} \\
 &= 6.21 \text{ m s}^{-2}
 \end{aligned}$$

The error on g is calculated using the rule for products, eq. (2.15):

$$\left(\frac{\Delta g}{g}\right)^2 = \left(\frac{\Delta m}{m}\right)^2$$

from which

$$\begin{aligned}
 \Delta g &= g \frac{\Delta m}{m} \\
 &= 4\pi^2 \frac{\Delta m}{m^2} \\
 &= \frac{4\pi^2(0.22 \text{ s}^2 \text{ m}^{-1})}{(6.36 \text{ s}^2 \text{ m}^{-2})^2} \\
 &= 4\pi^2(5.439 \times 10^{-3} \text{ m s}^{-2}) \\
 &= 0.2147 \text{ m s}^{-2}.
 \end{aligned}$$

Thus

$$g = (6.21 \pm 0.21) \text{ m s}^{-2}$$

From this result it is evident that the true value of g does not fall within the range of 6.00 m s^{-2} to 6.42 m s^{-2} determined by the experimental errors. We can therefore conclude that the experimenter made some mistake or that a systematic

error influenced the results.

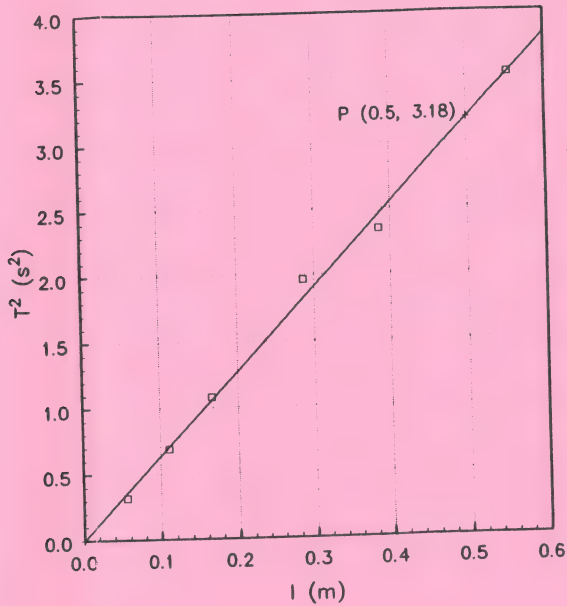


Figure 9: Graph of T^2 versus l

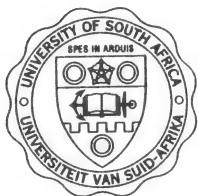
[20]

MRS J A CILLIERS

Tel.: 429-8018

UNISA

PHY103-8/202/95



96198354

UNISA **UNIVERSITY OF SOUTH AFRICA**

DEPARTMENT OF PHYSICS

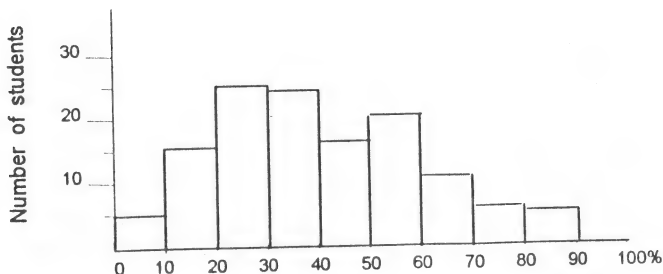
PHYSICS PHY103-8

TUTORIAL LETTER 105 – 1995

POST MORTEM FOR ASSIGNMENT 2

Dear Student,

I have included a histogram of the marks obtained by students for this assignment.



Problems 6.19, 7.19 and 8.23 which dealt with the processing and analysis of experimental data were poorly answered by most students.

It is clear from the assignments that there are a number of general principles which need to be pointed out.

1. **UNITS ARE IMPORTANT** and should not be neglected. This means that you have to include units in the tables you use for these solutions. For example, the first table for problem 6.19 should contain the following units:

x (m)	v_{0x} (10^{-2}m s^{-2})	v_x (m s^{-1})	t (s)
0.1	4.85	1.25	15.400
0.5	4.90	2.60	36.010
1.0	4.95	3.39	50.020
1.5	4.83	4.08	61.010
1.9	5.00	6.67	69.341
2.1	4.74	6.06	74.009
2.5	4.87	5.26	78.099

The quantities

$$D, m, c, s_m, s_c$$

usually also have units, which form part of your calculation and should be shown in the answers. In the linear regression analysis for the equation

$$v_x = (g \sin \theta)t + v_{0x}$$

in problem 6.19, D has the units s^2 , m has the units m s^{-2} and c has the units m s^{-1} . Likewise the units of s_m are m s^{-2} and for s_c they are m s^{-1} . In the key you will see that the calculations for D , m , c , s_m , and s_c do not only contain units in the answers, but in the interem steps as well. I would like to recommend this as a general practice - it is a valuable way of checking your calculations. If the units in the interem steps do not provide you with the correct units for the answer, you know that you have made a mistake somewhere.

Graphs should also contain units! By this I mean that you should include the units of each variable in brackets when you label the axes. When

plotting v_x versus t in problem 6.19, the y-axis represents v_x (m s^{-1}) and the x-axis presents $t(\text{s})$. WRITE DOWN v_x (m s^{-1}) as the label for the y-axis and $t(\text{s})$ as the label for the x-axis, as shown below.

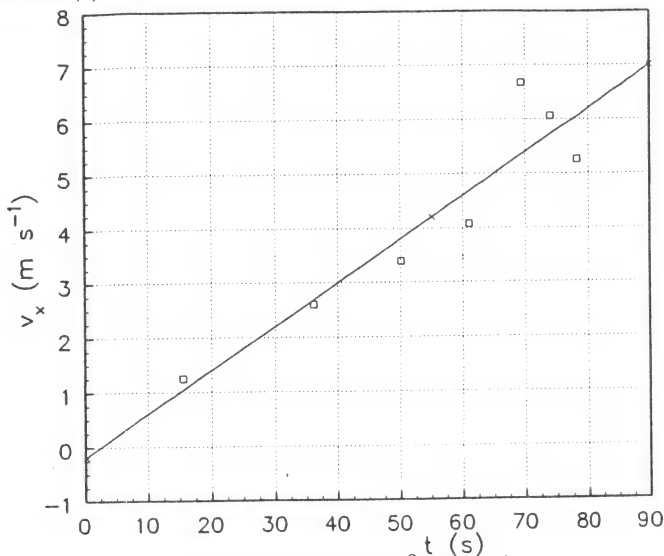


Figure 1: Graph of T^2 versus h

2. Many students go through the complete process of doing the linear regression analysis without ever writing down the equation under consideration. The first step for a problem like 6.19, 7.19 and 8.23 is to decide which graph you are going to plot. For this purpose you need to look at the EQUATION. WRITE DOWN THIS EQUATION and then modify it to the form

$$y = mx + c$$

For the first graph of problem 6.19 the equation is

$$\begin{aligned} v_x &= v_{0x} + g \sin \theta t \\ &= (g \sin \theta)t + v_{0x} \end{aligned}$$

which is already in the form $y = mx + c$.

It is very important that you state which physical quantity is presented by which variable, i.e.

$$y \equiv v_x, \quad x \equiv t, \quad m \equiv g \sin \theta \quad \text{and} \quad c \equiv v_{0x}.$$

This does not mean that you will ONLY plot straight line graphs. Sometimes an equation does not display a linear relation, and you can be asked to illustrate this by plotting the graph. An example of this is the graph of x versus t which depicts the quadratic equation

$$x = ut + \frac{1}{2}at^2.$$

3. I suggest that you read Chapter 3 on GRAPHS and Chapter 4 on PLANNING AN EXPERIMENT AND WRITING A REPORT very thoroughly before doing the next assignment.

Study example 3.5 carefully to see how you should do a linear regression analysis and 3.6 to see how to plot a graph. Also study the model report in section 4.4. This provides the correct approach to be followed for solving problems like 6.19, 7.19 and 8.23 as well as an additional example of a linear regression analysis and the subsequent plotting of a graph.

In addition to these general comments there are some problems which warrant specific comments. They are:

6.19

The value of $g = (10 \pm 2) \text{ m s}^{-2}$ which is found from the slope of the graph of v_x versus t is very close to the actual value. Unfortunately the data contains an error of magnitude which renders the result for g from the slope of the graph of v_x^2 versus s too large by a factor 100. No marks were therefore allocated for the graph of v_x^2 versus x , but 20 marks were allocated to the graph of v_x versus t and 4 marks for the graph of x versus t .

Take note that the latter is NOT A STRAIGHT LINE GRAPH! The instantaneous velocity at $t = 0$ is given by the slope of the tangent to the curve at $t = 0$, as is explained in the key.

7.12

The answer to this question is written out in great detail in the key. I recommend that you study this solution very well, as it provides you with the correct reasoning for analysing the influence of friction for this experiment.

Although a number of students answered this question very well, there is one misconception that was quite general. It concerns the direction in which the frictional force acts. Most students assumed this direction as opposite to the centripetal force for Experiment 2(a). As you can see from the key, the frictional

force is actually in the same direction as the centripetal force while the sphere is moving outward.

The important principle to remember, is that friction always opposes the MOTION, and its direction is therefore determined by the direction of the motion. When the apparatus is brought to a stop, the sphere moves back along the perspex tube, and at that stage the friction is indeed directed outward. From this you can see that the direction of the frictional force changed when the direction of the sphere's motion changed.

The key contains a typing error on p. 11. The expression on the third line of text below Figure 5 should read:

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0 : \vec{W} + \vec{N} = 0$$

7.19

A common mistake for this problem was that students failed to calculate the PERIOD

$$T = \frac{t}{30}$$

of the motion. The speed

$$v = \frac{2\pi R}{T}$$

was then calculated using the time for 30 revolutions instead of T , which made quite a mess of the graph!

Please correct the typing error in the table on p. 15 of the key. The heading of the third column of the table should be $T = t/30$.

8.2

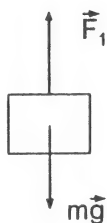
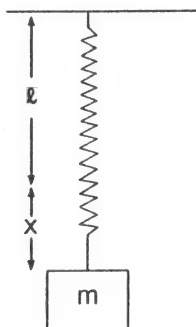
Very few students used the fact that the spring constants of two springs in series add reciprocally to give the reciprocal of the effective spring constant, i.e.

$$\frac{1}{k'} = \frac{1}{k_1} + \frac{1}{k_2}$$

and in this case $k_1 = k_2 = k$.

Because I am aware that neither the study guide nor PPB contains the relevant theory, I would like to offer the following explanation without using the effective spring constant.

First consider all the forces acting on a single spring and a mass m when the system is in equilibrium:



Forces on mass



Forces on spring

The mass experiences two forces; the weight $m\vec{g}$ downwards, and the upward force $\vec{F}_1$ exerted by the spring on the mass. Because the mass is in equilibrium

$$\vec{F}_1 = -m\vec{g} \quad (1)$$

(Is $\vec{F}_1$ and $m\vec{g}$ an action-reaction pair? Remember, they act on the SAME body!)

According to Newton's third law the mass exercises a force

$$\vec{F}_2 = -\vec{F}_1 \quad (2)$$

on the spring. (Is $\vec{F}_1$ and $\vec{F}_2$ an action-reaction pair? They act on DIFFERENT bodies!) The ceiling exerts a force $\vec{F}_3$ on the spring and if the mass of the spring is very small

$$\vec{F}_2 = -\vec{F}_3 \quad (3)$$

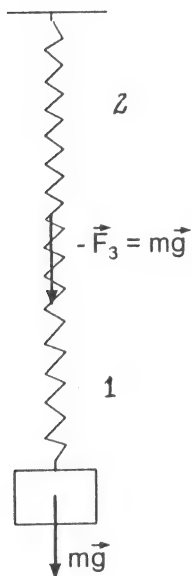
when the system is in equilibrium. From eqs. (1) and (2) it is clear that

$$\vec{F}_2 = m\vec{g}$$

and from (3) we can now deduce that

$$m\vec{g} = -\vec{F}_3.$$

When a second identical spring is inserted between the ceiling and the first spring, a force of magnitude $F_3 = mg$ will be exerted by spring 1 on spring 2.



Because a force mg caused an elongation of x in spring 1, the force $F_3 = mg$ will also cause spring 2 to extend by x . The total elongation is therefore $2x$.

8.16

The most important fact for this solution is that the period of a second pendulum

$$T = 2 \text{ s.}$$

As this was given, the solution is really very simple.

The key contains a typing error in the second to last line on p. 18. It should read

$$\text{or} \quad T^2 = \frac{4\pi^2}{g} \ell.$$

8.23

Unfortunately the table in the Study Guide giving the data for this problem contains two printing errors which make nonsense of the results! The unit for the length of the string should be cm and not m, and the time in the last row of the

table is for 80 oscillations and not 20.

No marks were allocated to this problem, and the total for the assignment was therefore 80 marks. The key contains the correct data and calculations. As an exercise you can redo the problem with the correct data and then evaluate your effort using the solution in the key.

MRS J A CILLIERS
Tel.: 429-8018

UNISA



96194677

UNISA UNIVERSITY OF SOUTH AFRICA

DEPARTMENT OF PHYSICS

PHYSICS PHY103-8

TUTORIAL LETTER 203 – 1995

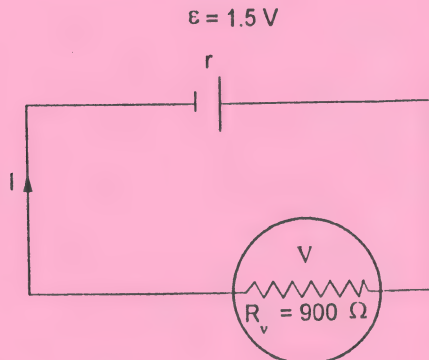
KEY TO ASSIGNMENT 3

9.2

The current through a linear circuit element is linearly dependent on the potential difference applied across this circuit element.

For a non-linear circuit element, the relationship between current and potential difference is not linear. [4]

9.19



When Kirchhoff's loop rule is applied to the circuit, it renders

$$\begin{aligned}\Sigma \varepsilon - \Sigma IR &= 0 \\ \text{i.e.} \quad \varepsilon &= IR_V + Ir \\ \text{or} \quad r &= \frac{\varepsilon}{I} - R_V\end{aligned}\quad (1)$$

where r is the internal resistance of the battery and I is the current through the circuit.

A voltmeter reading of 1.35 V implies that

$$IR_V = 1.35\text{ V}$$

from which

$$\begin{aligned}I &= \frac{1.35\text{ V}}{900\ \Omega} \\ &= 1.5 \times 10^{-3}\text{ A}.\end{aligned}$$

Substituting this value into eq. (1) renders

$$\begin{aligned}r &= \frac{1.5\text{ V}}{1.5 \times 10^{-3}\text{ A}} - 900\ \Omega \\ &= 100\ \Omega\end{aligned}$$

[9]

9.21

According to Ohm's law

$$V = IR$$

For this experiment I was measured as a function of V , so I is the *dependent* variable and V is the *independent* variable. We therefore rearrange the equation to read

$$I = \frac{1}{R} V$$

which is in the form of the straight line equation

$$y = mx + c$$

with

$$y \equiv I, \ x \equiv V, \ m \equiv \frac{1}{R} \text{ and } c = 0.$$

The summation needed for a linear regression analysis are tabulated below:

i	x_i (V)	y_i (A)	$x_i y_i$ (V A)	x_i^2 (V ²)	$m x_i + c$ (A)	δ_i^2 (10 ⁻³ A ²)
1	1	0.11	0.11	1	0.104	0.036
2	2	0.19	0.38	4	0.204	0.196
3	3	0.33	0.99	9	0.304	0.676
4	4	0.41	1.64	16	0.404	0.036
5	5	0.47	2.35	25	0.504	1.156
6	6	0.59	3.54	36	0.604	0.196
7	7	0.73	5.11	49	0.704	0.676
8	8	0.80	6.40	64	0.804	0.016
9	9	0.93	8.37	81	0.904	0.676
10	10	0.99	9.90	100	1.004	0.196
$\Sigma_{i=1}^{10}$	55	5.55	38.79	385		3.860

$$\begin{aligned}
 D &= n \Sigma x^2 - (\Sigma x)^2 \\
 &= (10)(385 \text{ V}^2) - (55 \text{ V})^2 \\
 &= 825 \text{ V}^2
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m &= \frac{n \Sigma xy - \Sigma x \Sigma y}{D} \\
 &= \frac{10(38.79 \text{ V A}) - (55 \text{ V})(5.55 \text{ A})}{825 \text{ V}^2} \\
 &= 0.100 \text{ A V}^{-1}
 \end{aligned}$$

and the y-intercept is

$$\begin{aligned}
 c &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{D} \\
 &= \frac{(385 \text{ V}^2)(5.55 \text{ A}) - (55 \text{ V})(38.79 \text{ V A})}{825 \text{ V}^2} \\
 &= 0.004 \text{ A.}
 \end{aligned}$$

The error on m is

$$\begin{aligned}s_m &= \left[\left(\frac{n}{n-2} \right) \frac{\Sigma \delta^2}{D} \right]^{\frac{1}{2}} \\&= \left[\frac{(10)(3.86 \times 10^{-3} \text{ A}^2)}{(8)(825 \text{ V}^2)} \right]^{\frac{1}{2}} \\&= 2.418 \times 10^{-3} \text{ A V}^{-1}\end{aligned}$$

and the error on c is

$$\begin{aligned}s_c &= \left[\frac{1}{n-2} \frac{\Sigma \delta^2 \Sigma x^2}{D} \right]^{\frac{1}{2}} \\&= \left[\frac{(3.86 \times 10^{-3} \text{ A}^2)(385 \text{ V}^2)}{(8)(825 \text{ V}^2)} \right]^{\frac{1}{2}} \\&= 1.501 \times 10^{-2} \text{ A}\end{aligned}$$

The slope is therefore

$$m = (0.100 \pm 0.002) \text{ A V}^{-1}$$

and the y-intercept

$$c = (0.004 \pm 0.015) \text{ A}$$

The resistance follows from the slope since

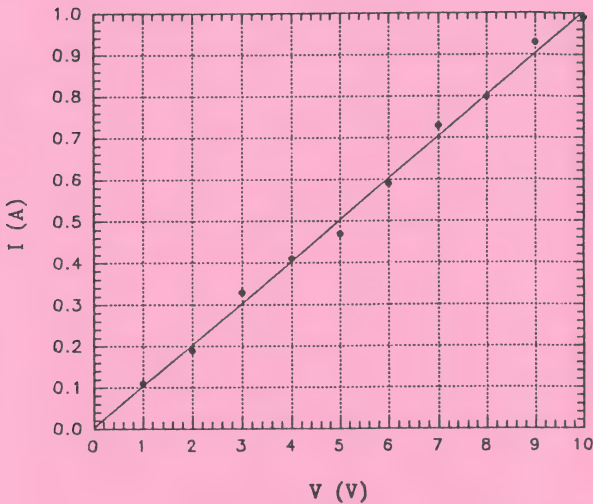
$$\begin{aligned}m &= \frac{1}{R} \\ \Rightarrow R &= \frac{1}{m} \\&= \frac{1}{0.100 \text{ A V}^{-1}} \\&= 10.0 \text{ V A}^{-1} \\&= 10.0 \Omega.\end{aligned}$$

The error on R is given by

$$\begin{aligned}\frac{\Delta R}{R} &= \frac{\Delta m}{m} \\ \Rightarrow \Delta R &= R \frac{\Delta m}{m} \\&= \frac{(10.0 \Omega)(0.002 \text{ A V}^{-1})}{0.100 \text{ A V}^{-1}} \\&= 0.2 \Omega.\end{aligned}$$

The resistance is therefore

$$R = (10.0 \pm 0.2) \, \Omega$$



Graph of I versus V

[25]

10.4

The minimum amount of information needed to determine the direction of the magnetic force on a charged particle is

- the direction of the magnetic field
- the direction in which the charge is moving
- whether the charge is positive or negative

[3]

10.12

Simplistically stated, we could say that the reaction of force $\vec{F}$ is the force exerted by the electron on the magnetic field, but we need to think carefully about what this actually means.

A moving charge sets up a magnetic field in the space around it. The reaction to force $\vec{F}$ is therefore the result of the interaction between the electron's magnetic field and the magnetic field through which it is moving. [2]

10.23

The magnetic field between the Helmholtz coils causes the electrons to move along a circular path of radius r . The radius follows from the relation

$$y = \frac{1}{2r}(x^2 + y^2). \quad (10.16)$$

This equation is of the form

$$y' = mx' + c$$

where

$$y' \equiv y, \quad x' \equiv x^2 + y^2, \quad m \equiv \frac{1}{2r} \text{ and } c = 0.$$

If a graph of y versus $x^2 + y^2$ renders a straight line, it will be a confirmation of eq. (10.16) and we would then be able to conclude that the electrons move in a straight line.

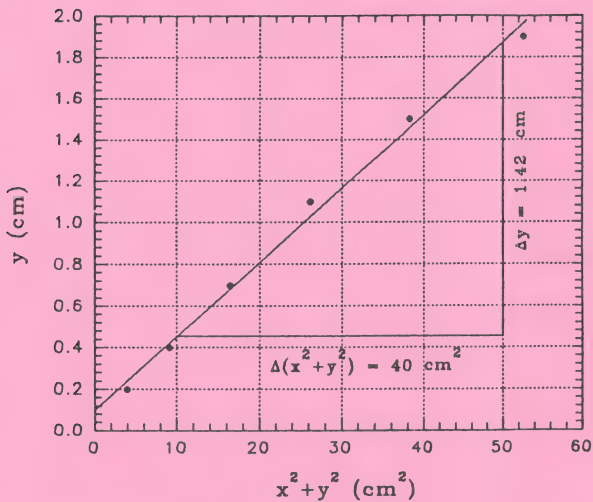
x (cm)	y (cm)	$x^2 + y^2$ (cm^2)
2.0	0.2	4.04
3.0	0.4	9.16
4.0	0.7	14.49
5.0	1.1	26.21
6.0	1.5	38.25
7.0	1.9	52.61

The slope of the graph is

$$\begin{aligned}
 m &= \frac{\Delta y}{\Delta(x^2 + y^2)} \\
 &= \frac{1.42 \text{ cm}}{40 \text{ cm}^2} \\
 &= 3.55 \times 10^{-2} \text{ cm}^{-1}.
 \end{aligned}$$

The radius of curvature follows from the slope because

$$\begin{aligned}
 m &= \frac{1}{2r} \\
 \Rightarrow r &= \frac{1}{2m} \\
 &= \frac{1}{2(3.55 \times 10^{-2} \text{ cm}^{-1})} \\
 &= 14.1 \text{ cm}.
 \end{aligned}$$



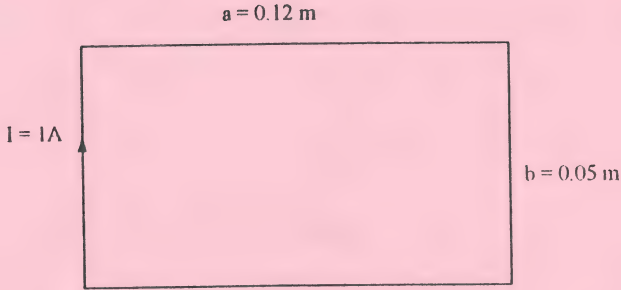
Graph of x versus $x^2 + y^2$

[20]

REMARK:

A linear regression analysis was not done on these results, because the quantity plotted on the x-axis contained both x and y .

11.5



The torque on a coil of area A , carrying a current I in a field of magnetic induction B , is given by

$$\vec{\tau} = I A \hat{n} \times \vec{B}$$

so the magnitude of the torque is

$$\tau = IAB \sin \theta$$

where θ is the angle between the normal to the coil and the magnetic field $\vec{B}$. From this expression it is clear that the maximum value of τ occurs when $\sin \theta$ is at a maximum, i.e. when $\sin \theta = 1$ ($\theta = \frac{n\pi}{2}$, $n = 1, 3, 5, \dots$).

Thus

$$\begin{aligned} \tau_{\max} &= IAB \\ &= I(ab)B \\ &= (1 \text{ A})(0.12 \text{ m})(0.05 \text{ m})(0.10 \text{ N A}^{-1} \text{ m}^{-1}) \\ &= 6 \times 10^{-4} \text{ N m.} \end{aligned}$$

[c.]

11.14

$$\vec{F} = q\vec{E} + (q\vec{v} \times \vec{B})$$

$\vec{F}$ represents the resultant force on a charge q moving with velocity $\vec{v}$ through a region in which an electric field $\vec{E}$ and a magnetic field $\vec{B}$ are present.

$q\vec{E}$ represents the force exerted on the charge by the electric field. The direction of this force is the same as that of the electric field in the case of a positive charge, and opposite to the direction of the field for a negative charge.

$q\vec{v} \times \vec{B}$ represents the force exerted on the charge by the magnetic field. The direction of this force is perpendicular to both the velocity $\vec{v}$ and the magnetic field $\vec{B}$ and is given by the right hand rule. [7]

11.16

The relationship between the length L of the horizontal wire and the force $\vec{F}$ is given by

$$F = ILB \quad (11.8)$$

which is of the form

$$y = mx + c$$

with

$$y \equiv F, \quad x \equiv L, \quad m \equiv IB \text{ and } c = 0.$$

The magnitude of the force on the horizontal wire for each value of d can be calculated from eq. (10.8) using the given values for m , g and L_0 :

$$\begin{aligned} F &= \frac{mgd}{L_0} \\ &= \frac{(0.001 \text{ kg})(9.8 \text{ m s}^{-2})d}{(0.10 \text{ m})} \\ &= (0.098 \text{ kg s}^{-2})d \end{aligned}$$

L (m)	d (m)	F (10^{-3} N)
0.02	0.0077	0.75
0.04	0.0152	1.49
0.06	0.0233	2.28
0.08	0.0320	3.14

The summations required for the linear regression analysis are tabulate below:

i	x_i (m)	y_i (10^{-3} N)	$x_i y_i$ (10^{-4} N m)	x_i^2 (10^{-3} m ²)	$m x_i + c$ (10^{-3} N)	δ_i^2 (10^{-9} N)
1	0.02	0.75	0.150	0.4	0.721	0.041
2	0.04	1.49	0.596	1.6	1.517	0.729
3	0.06	2.28	1.368	3.6	2.313	1.089
4	0.08	3.14	2.512	6.4	3.109	0.961
$\Sigma_{i=1}^4$	0.20	7.66	4.626	12.0		3.620

$$\begin{aligned}
 D &= n \Sigma x^2 - (\Sigma x)^2 \\
 &= (4)(12.0 \times 10^{-3} \text{ m}^2) - (0.20 \text{ m})^2 \\
 &= 8.0 \times 10^{-3} \text{ m}^2
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m &= \frac{n \Sigma xy - \Sigma x \Sigma y}{D} \\
 &= \frac{(4)(4.626 \times 10^{-4} \text{ N m}) - (0.20 \text{ m})(7.66 \times 10^{-3} \text{ N})}{(8.0 \times 10^{-3} \text{ m}^2)} \\
 &= 3.98 \times 10^{-2} \text{ N m}^{-1}
 \end{aligned}$$

and the y-intercept is

$$\begin{aligned}
 c &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{D} \\
 &= \frac{(12.0 \times 10^{-3} \text{ m}^2)(7.66 \times 10^{-3} \text{ N}) - (0.20 \text{ m})(4.626 \times 10^{-4} \text{ N m})}{(8.0 \times 10^{-3} \text{ m}^2)} \\
 &= -7.50 \times 10^{-5} \text{ N}.
 \end{aligned}$$

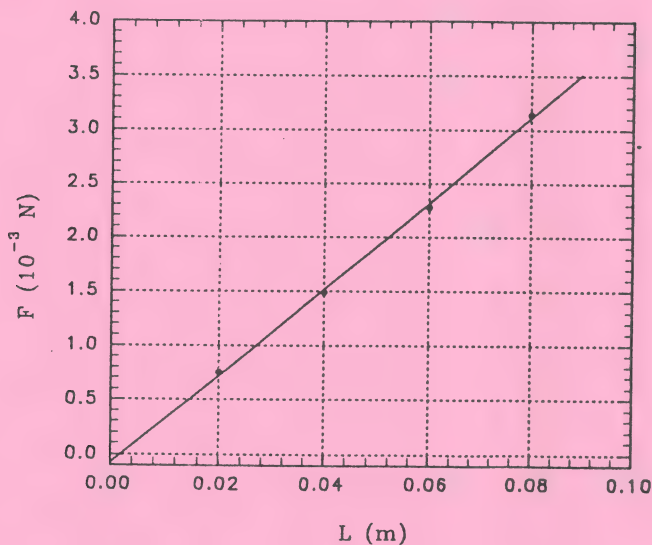
The error on the slope is

$$\begin{aligned}
 s_m &= \left[\frac{n}{n-2} \frac{\Sigma \delta^2}{D} \right]^{\frac{1}{2}} \\
 &= \left[\frac{(4)(3.620 \times 10^{-9} \text{ N})}{(2)(8.0 \times 10^{-3} \text{ m}^2)} \right]^{\frac{1}{2}} \\
 &= 9.51 \times 10^{-4} \text{ N m}^{-1}
 \end{aligned}$$

and the error on the y-intercept is

$$\begin{aligned}
 s_c &= \left[\frac{\Sigma \delta^2 \Sigma x^2}{(n-2)D} \right]^{\frac{1}{2}} \\
 &= \left[\frac{(3.62 \times 10^{-9} \text{ N})(12.0 \times 10^{-3} \text{ m})}{(2)(8.0 \times 10^{-3} \text{ m}^2)} \right]^{\frac{1}{2}} \\
 &= 5.21 \times 10^{-5}.
 \end{aligned}$$

The graph of F versus L is a straight line, which illustrates that $F \propto L$. (We say that F is linearly dependent on L).



Graph of F versus L

The value of B follows from the slope, because

$$\begin{aligned}
 m &= IB \\
 \Rightarrow B &= \frac{m}{I} \\
 &= \frac{3.98 \times 10^{-2} \text{ N m}^{-1}}{1.2 \text{ A}} \\
 &= 3.317 \times 10^{-2} \text{ T}.
 \end{aligned}$$

The error on B follows from eq. (2.10) as

$$\Delta B = \frac{\Delta m}{I}$$

since I can be regarded as a constant. Thus

$$\begin{aligned} s_B &= \frac{s_m}{I} \\ &= \frac{9.51 \times 10^{-4} \text{ N m}^{-1}}{1.2 \text{ A}} \\ &= 7.93 \times 10^{-4} \text{ T} \end{aligned}$$

so

$$\begin{aligned} B &= (3.317 \pm 0.079) \times 10^{-2} \text{ T} \\ &\simeq (3.3 \pm 0.1) \times 10^{-2} \text{ T}. \end{aligned}$$

[25]

MRS J A CILLIERS
Tel.: 429-8018

UNISA

PHY103-8/203/95

NB! Take note:

Radio Unisa 1995

STUDENT SERVICES BUREAU

Tel: (012)429-3513

The SABC has agreed to broadcast the following study motivational and student supportive programmes of the Student Services Bureau (also refer to your Year Planner).

Tune in to Radio 2000 on the following Sunday evenings:

SDB 01 Motivation and planning	25/06/95 20:00-21:00
SDB 02 Assignments	02/07/95 20:00-21:00
SDB 03 Reading skills	09/07/95 20:00-21:00
SDB 04 Writing skills	16/07/95 20:00-21:00
SDB 05 Study techniques	23/07/95 20:00-21:00
SDB 06 Stress management	30/07/95 20:00-21:00
SDB 07 Exam preparation	06/08/95 20:00-21:00



96223316

UNISA UNIVERSITY OF SOUTH AFRICA

DEPARTMENT OF PHYSICS

PHYSICS PHY103-8

TUTORIAL LETTER 204 – 1995

KEY FOR ASSIGNMENT 4

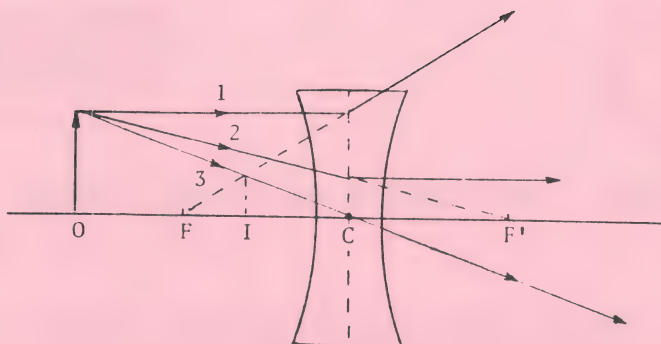
CONTENTS:

1. Key to Assignment 4
2. Post Mortem for Assignment 3

12.3

To determine the focal length of a thin concave lens graphically, the following useful rules should be kept in mind

- a) A ray parallel to the axis of the lens passes through the focal point of a converging lens after refraction by the lens, or appears to come from the focal point of a diverging lens.
- b) A ray through the focal point emerges parallel to the axis of the lens.
- c) A ray through the centre of the lens is not deviated, because the two lens surfaces through which the ray passes are very nearly parallel at that point for a thin lens.



In the diagram F and F' are the focal points of the lens, and O is the object.

Ray 1 leaves the object O parallel to the axis of the lens and appears to come from the focal point F of the lens.

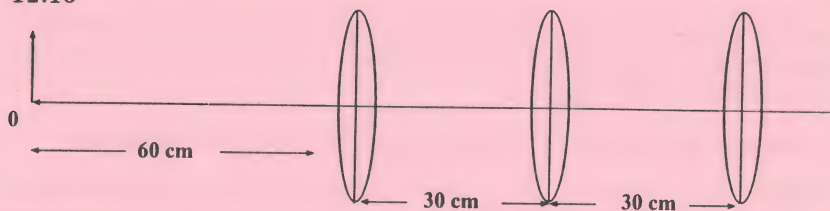
Ray 2 is initially directed toward the far focal point F' and is refracted to emerge parallel to the axis of the lens.

Ray 3 passes undeviated through the lens and intersects ray 1 to form an image at I .

From the diagram it is clear that the image is virtual, upright and diminished. If the diagram is drawn to scale, the position and size of the image can be measured.

[8]

12.10



First find the image formed by the first lens, for which

$$s = +60 \text{ cm}$$

$$f = +20 \text{ cm}$$

(The signs for these distances are allocated according to the sign convention of p .)

234 in the study guide).

From the thin lens equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \quad (12.12)$$

the image distance s' is given by

$$\begin{aligned} \frac{1}{s'} &= \frac{1}{f} - \frac{1}{s} \\ &= \frac{1}{20 \text{ cm}} - \frac{1}{60 \text{ cm}} \\ &= \frac{3 - 1}{60 \text{ cm}} \end{aligned}$$

so that

$$s' = 30 \text{ cm}.$$

This means that an image is formed by the first lens at a distance 30 cm to the right of the lens, i.e. exactly at the centre of the second lens.

Equation (12.12) can be rewritten as

$$\frac{s' + s}{ss'} = \frac{1}{f}$$

so

$$(s + s')f = ss'.$$

For the second lens the object distance s is zero, so

$$(0 + s')f = 0$$

from which

$$s' = 0.$$

The object for the third lens is therefore situated 30 cm to the left of the lens, so

$$s = +30 \text{ cm}$$

$$f = +20 \text{ cm}.$$

Substitution of these values into eq. (12.12) renders

$$\frac{1}{30 \text{ cm}} + \frac{1}{s'} = \frac{1}{20 \text{ cm}}$$

from which

$$\begin{aligned}\frac{1}{s'} &= \frac{1}{20 \text{ cm}} - \frac{1}{30 \text{ cm}} \\ &= \frac{3 - 2}{60 \text{ cm}}\end{aligned}$$

so that

$$s' = + 60 \text{ cm}.$$

As s' is positive, we can conclude that the final image is formed at 60 cm to the right of the third lens. [10]

12.26

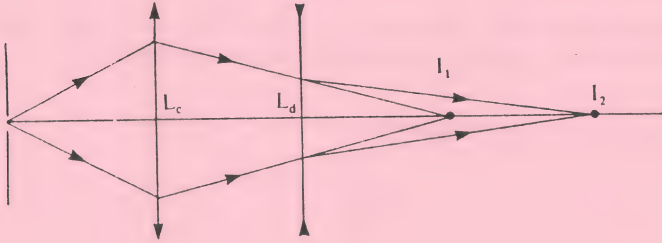


Figure 3: Concave and convex lens system.

Figure 3 shows the experimental setup. The image I_1 is formed by the convex lens L_c and serves as an object for the concave lens L_d which forms the final image I_2 . The object is at the zero position, so the object distance for the concave lens

$$s = L_d - I_1$$

and the image distance for the concave lens

$$s' = I_2 - L_d.$$

The values of s and s' are tabulated below.

L_d (m)	I_1 (m)	I_2 (m)	s (m)	s' (m)
0.580	0.627	0.650	-0.047	0.070
0.500	0.530	0.538	-0.030	0.038
0.400	0.492	0.640	-0.092	0.240
0.280	0.422	2.90	-0.142	2.620
0.281	0.395	0.753	-0.114	0.472
0.382	0.460	0.550	-0.078	0.168
0.501	0.574	0.650	-0.073	0.149

The relation between object distance s , image distance s' and focal length f is given by the thin lens equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

which can be rewritten as

$$\frac{1}{s'} = -\frac{1}{s} + \frac{1}{f}$$

which is of the form

$$y = mx + c$$

with

$$y \equiv \frac{1}{s'}, \quad x \equiv \frac{1}{s}, \quad m = -1 \quad \text{and} \quad c \equiv \frac{1}{f}$$

Note that we have taken $\frac{1}{s'}$ to be the DEPENDENT variable, because s' is measured as a function of s in the experiment.

The values of $\frac{1}{s}$ and $\frac{1}{s'}$ and the summations needed for the linear regression analysis are tabulated below.

i	$x \equiv \frac{1}{s}$ (m ⁻¹)	$y \equiv \frac{1}{s'}$ (m ⁻¹)	$x_i y_i$ (m ⁻²)	x_i^2 (m ⁻²)	$m x_i + c$ (m ⁻¹)	δ_i^2 (m ⁻²)
1	-21.28	14.29	-304.09	452.84	14.40	0.0121
2	-33.33	26.32	-877.25	1110.89	26.33	0.0001
3	-10.87	4.17	-45.33	116.21	4.09	0.0064
4	-7.04	0.38	-2.68	49.56	0.30	0.0064
5	-8.77	2.12	-18.59	76.91	2.01	0.0121
6	-12.82	5.95	-76.28	164.44	6.02	0.0049
7	-13.70	6.71	-91.92	187.69	6.89	0.0324
Σ	-107.81	59.94	-1416.14	2158.85		0.0744

$$\begin{aligned}
 D &= n\Sigma x^2 - (\Sigma x)^2 \\
 &= 7(2158.85 \text{ m}^{-2}) - (-107.81 \text{ m}^{-1})^2 \\
 &= 3.49 \times 10^3 \text{ m}^{-2}
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m &= \frac{n\Sigma xy - \Sigma x \Sigma y}{D} \\
 &= \frac{(7)(-1416.14 \text{ m}^{-2}) - (-107.81 \text{ m}^{-1})(59.94 \text{ m}^{-1})}{3.49 \times 10^3 \text{ m}^{-2}} \\
 &= -0.99
 \end{aligned}$$

[Note that in this case the slope does not have units, because the units (m⁻²) cancel]

The y-intercept

$$\begin{aligned}
 c &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{D} \\
 &= \frac{(2158.85 \text{ m}^{-2})(59.94 \text{ m}^{-1}) - (-107.81 \text{ m}^{-1})(-1416.14 \text{ m}^{-2})}{3.49 \times 10^3 \text{ m}^{-2}} \\
 &= -6.67 \text{ m}^{-1}
 \end{aligned}$$

The error on the slope

$$s_m = \left[\frac{n\Sigma \delta^2}{(n-2)D} \right]^{\frac{1}{2}}$$

$$\begin{aligned}
 &= \left[\frac{(7)(7.44 \times 10^{-2} \text{ m}^{-2})}{(5)(3.49 \times 10^3 \text{ m}^{-2})} \right]^{\frac{1}{2}} \\
 &= 0.0055
 \end{aligned}$$

and the error on the y-intercept is

$$\begin{aligned}
 s_c &= \left[\frac{\Sigma \delta^2 \Sigma x^2}{(n-2)D} \right]^{\frac{1}{2}} \\
 &= \left[\frac{(7.44 \times 10^{-2} \text{ m}^{-2})(2158.85 \text{ m}^{-2})}{(5)(3.49 \times 10^3 \text{ m}^{-2})} \right]^{\frac{1}{2}} \\
 &= 0.096 \text{ m}^{-1}
 \end{aligned}$$

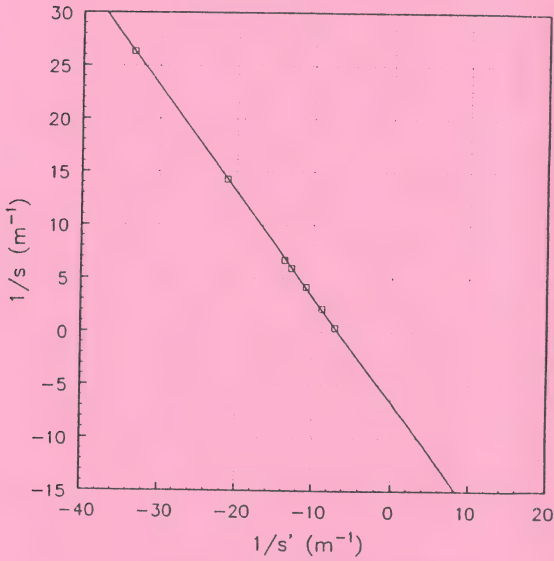


Figure 4: Graph of $\frac{1}{s'}$ versus $\frac{1}{s}$.

$$c \equiv \frac{1}{f}$$

the focal length

$$\begin{aligned}
 f &= \frac{1}{c} \\
 &= -0.15 \text{ m} \\
 &= -15 \text{ cm.}
 \end{aligned}$$

(The focal length of a concave lens is a negative quantity and therefore the lens is also known as a negative lens).

The error on f follows from

$$\frac{\Delta f}{f} = \frac{\Delta c}{c}$$

or

$$\begin{aligned} s_f &= f \frac{s_c}{c} \\ &= \frac{(-0.15 \text{ m})(0.096 \text{ m}^{-1})}{(-6.67 \text{ m}^{-1})} \\ &= 0.002 \text{ m} \\ &= 0.2 \text{ cm} \end{aligned}$$

Thus

$$f = (15.0 \pm 0.2) \text{ cm}.$$

[20]

13.4

By 1924 it was accepted that light possesses both wave and particle properties. De Broglie argued that, since light has a dual character this duality should also be true for matter. Matter particles, therefore also have wave properties. He postulated that the wavelength associated with a material particle is

$$\lambda = \frac{h}{mv}$$

where m is the mass of the particle, v is its velocity and h is Planck's constant. This wavelength is known as the *de Broglie wavelength* of a particle. [2]

13.15

The positions of the maxima formed by the diffraction grating are given by

$$a \sin \theta = n\lambda, \quad n = 0, \pm 1, \pm 2, \dots \quad (13.8)$$

where a is the distance between two adjacent slits. It is clear from this equation, that the sines of the deviation angles of the maxima are proportional to the ratio λ/a . The number of maxima that can be observed, is limited by the fact that $\sin \theta$ cannot be greater than 1 (or smaller than -1).

For $N = 364$ slits per cm the grating spacing

$$\begin{aligned} a &= \frac{1}{N} \\ &= \frac{1}{364 \text{ cm}^{-1}} \\ &= 2.75 \times 10^{-3} \text{ cm} \\ &= 2.75 \times 10^{-5} \text{ m} \end{aligned}$$

The given wavelength is $\lambda = 550 \times 10^{-9} \text{ m}$, so

$$\begin{aligned} \frac{\lambda}{a} &= \frac{5.50 \times 10^{-7} \text{ m}}{2.75 \times 10^{-5} \text{ m}} \\ &= 0.02 \end{aligned}$$

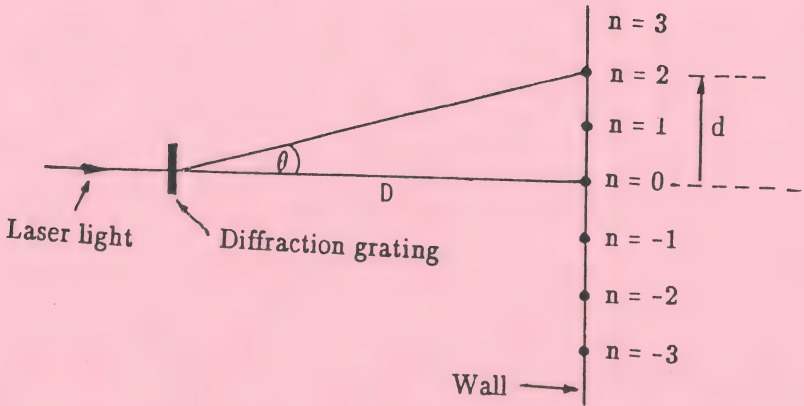
Looking at the light source through the grating, the "straight ahead" source will be the central maximum. For this grating we have

$$\sin \theta = n \frac{\lambda}{d} = n(0.02)$$

so the observer should be able to see images for $n = 0 \pm 1, \pm 2, \dots, \pm 50$. Counting images, the observer should be able to see 101 images of the same source (50 on either side of the central maximum) plus the central image, making 101 in all.

Under normal circumstances one is usually only able to see three or four orders (i.e. $n = 0, \pm 1, \pm 2, \pm 3$). The number of slits given for this grating should probably be 364 per mm and not per cm, which would give the more realistic answer of 11 images in all.

13.18



The figure above shows the positions of the interference maxima produced by the diffraction grating.

The grating contains

$$N = 2.953 \times 10^5 \text{ slits m}^{-1}$$

so the distance between two adjacent slits is

$$\begin{aligned} a &= \frac{1}{N} \\ &= \frac{1}{2.953 \times 10^5 \text{ slits m}^{-1}} \\ &= 3.386 \times 10^{-6} \text{ m.} \end{aligned}$$

The position of the maxima are determined by the equation

$$a \sin \theta = n\lambda, \quad n = 0, \pm 1, \pm 2, \quad (13.8)$$

which can be rewritten as

$$\sin \theta = \frac{\lambda}{a} n$$

to represent a straight line with

$$y \equiv \sin \theta, \quad x \equiv n, \quad m \equiv \frac{\lambda}{a} \quad \text{and} \quad c = 0.$$

The angle θ can be determined for each maximum by using the fact that

$$\tan \theta = \frac{d}{D}$$

as can be seen from the figure. Thus

$$\theta = \tan^{-1} \left(\frac{d}{D} \right)$$

and with $D = 60$ cm, the results are as tabulated below

n	d (cm)	d/D	θ	$\sin \theta$
-3	-43.1	-0.718	-35.68°	-0.583
-2	-25.5	-0.425	-23.03°	-0.391
-1	-12.0	-0.200	-11.31°	-0.196
0	0.0	0.000	0.00	0.000
1	12.3	0.205	11.59°	0.201
2	26.2	0.437	23.61°	0.401
3	45.4	0.758	37.16°	0.604

The summations needed for the linear regression analysis are tabulated below:

i	x_i	y_i	$x_i y_i$	x_i^2	$m x_i + c$	δ_i^2 (10^{-5})
1	-3	-0.583	1.749	9	-0.589	3.6
2	-2	-0.391	0.782	4	-0.391	0.0
3	-1	-0.196	0.196	1	-0.193	0.9
4	0	0.000	0.000	0	0.005	2.5
5	1	0.201	0.201	1	0.203	0.4
6	2	0.401	0.802	4	0.401	0.0
7	3	0.604	1.812	9	0.599	2.5
Σ	0	0.036	5.542	28		9.9

$$\begin{aligned}
 D &= n\Sigma x^2 - (\Sigma x)^2 \\
 &= (7)(28) - 0 \\
 &= 196
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m &= \frac{n\Sigma xy - \Sigma x\Sigma y}{D} \\
 &= \frac{(7)(5.542) - (0)(0.036)}{196} \\
 &= 0.198
 \end{aligned}$$

and the y-intercept is

$$\begin{aligned}
 c &= \frac{\Sigma x^2\Sigma y - \Sigma x\Sigma xy}{D} \\
 &= \frac{(28)(0.036) - (0)(5.542)}{196} \\
 &= 5.14 \times 10^{-3}.
 \end{aligned}$$

(Note that in this case D , m and c do not have units. THIS IS ONLY TRUE FOR THE SPECIFIC PROBLEM. Generally D , m and c DO HAVE UNITS!)

The error on the slope is

$$\begin{aligned}
 s_m &= \left[\frac{n\Sigma \delta^2}{(n-2)D} \right]^{\frac{1}{2}} \\
 &= \left[\frac{(7)(9.9 \times 10^{-5})}{(5)(196)} \right]^{\frac{1}{2}} \\
 &= 8.41 \times 10^{-4}
 \end{aligned}$$

and the error on the y-intercept is

$$\begin{aligned}
 s_c &= \left[\frac{\Sigma \delta^2 \Sigma x^2}{(n-2)D} \right]^{\frac{1}{2}} \\
 &= \left[\frac{(9.9 \times 10^{-5})(28)}{(5)(196)} \right]^{\frac{1}{2}} \\
 &= 1.68 \times 10^{-3}
 \end{aligned}$$

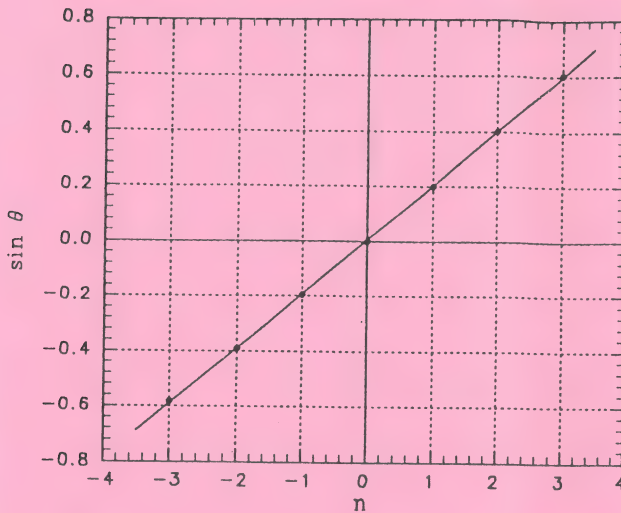


Figure 5: Graph of $\sin \theta$ versus n .

The wavelength λ is obtained from the slope

$$m = \frac{\lambda}{a}$$

so

$$\begin{aligned}\lambda &= ma \\ &= (0.198)(3.386 \times 10^{-6} \text{ m}) \\ &= 6.7043 \times 10^{-7} \text{ m}.\end{aligned}$$

The error on λ is given by

$$\begin{aligned}\Delta\lambda &= \Delta ma \\ &= (8.41 \times 10^{-4})(3.386 \times 10^{-6} \text{ m}) \\ &= 2.85 \times 10^{-9} \text{ m}\end{aligned}$$

Thus the wavelength of the laserbeam is

$$\lambda = (6.70 \pm 0.03) \times 10^{-7} \text{ m}.$$

14.6

The threshold frequency f_0 is the lowest frequency at which electrons are emitted. It is characteristic of the material being illuminated and at any frequency below f_0 no electrons will be emitted. [2]

14.18

The maximum kinetic energy of an emitted electron can have, is

$$E = hf - \phi$$

or

$$E = \frac{hc}{\lambda} - \phi \quad (14.4)$$

The work function of the surface is given by

$$\begin{aligned} \phi &= hf_0 \\ &= \frac{hc}{\lambda_0} \end{aligned}$$

Substituting this expression for ϕ into eqn. (14.4) renders

$$E = hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_0} \right)$$

and the maximum kinetic energy of electrons that are ejected by light of wavelength 1.24×10^{-7} m is therefore

$$\begin{aligned} E &= (6.63 \times 10^{-36} \text{ J s})(3 \times 10^8 \text{ m s}^{-1}) \left(\frac{1}{1.24 \times 10^{-7} \text{ m}} - \frac{1}{2.93 \times 10^{-7} \text{ m}} \right) \\ &= 9.25 \times 10^{-19} \text{ J.} \end{aligned}$$

Since this is kinetic energy, we can say that

$$\frac{1}{2} m_e v^2 = E$$

and thus the maximum velocity with which electrons escape from the metal is

$$\begin{aligned} v &= \left[\frac{2E}{m_e} \right]^{\frac{1}{2}} \\ &= \left[\frac{(2)(9.25 \times 10^{-19} \text{ J})}{(9.11 \times 10^{-31} \text{ kg})} \right]^{\frac{1}{2}} \\ &= 1.4 \times 10^6 \text{ m s}^{-1} \end{aligned}$$

14.24

The so-called *photoelectric equation*

$$eV = \frac{hc}{\lambda} - \phi \quad (14.4)$$

can be rewritten as

$$V = \frac{hc}{e} \frac{1}{\lambda} - \frac{\phi}{e}$$

which is of the form $y = mx + c'$ with

$$y \equiv V, \quad x \equiv \frac{1}{\lambda}, \quad m \equiv \frac{hc}{e} \quad \text{and} \quad c' = \frac{\phi}{e}.$$

The stopping potential was determined using transparencies which transmit a range of wavelengths. We will use the midvalue of the given wavelength band in each case. The results are tabulated below

λ (10^{-7} m)	$1/\lambda$ (10^7 m $^{-1}$)	V (volt)
0.9	1.11	12.4
1.5	0.67	7.5
3.0	0.33	3.0
4.0	0.25	2.2
6.0	0.17	1.1
8.0	0.13	0.57
10.0	0.10	0.20

The summations needed for the linear regression analysis are translated below.

i	x_i (10^7 m^{-1})	y_i (V)	$x_i y_i$ (10^7 V m^{-1})	x_i^2 (10^{13} m^{-2})	$m x_i + c$ (V)	δ_i^2 (10^{-2} V^2)
1	1.11	12.4	13.764	12.321	12.56	2.56
2	0.67	7.5	5.025	4.489	7.21	8.41
3	0.33	3.0	0.990	1.089	3.07	0.49
4	0.25	2.2	0.550	0.625	2.10	1.00
5	0.17	1.1	0.187	0.289	1.12	0.04
6	0.13	0.57	0.074	0.169	0.64	0.49
7	0.10	0.20	0.020	0.100	0.27	0.49
Σ	2.76	26.97	20.610	19.082		13.48

$$\begin{aligned}
 D &= n\Sigma x^2 - (\Sigma x)^2 \\
 &= (7)(19.082 \times 10^{13} \text{ m}^{-2}) - (2.76 \times 10^7 \text{ m}^{-1})^2 \\
 &= 5.74 \times 10^{14} \text{ m}^{-2}
 \end{aligned}$$

The slope is

$$\begin{aligned}
 m &= \frac{n\Sigma xy - \Sigma x \Sigma y}{D} \\
 &= \frac{(7)(20.610 \times 10^7 \text{ V m}^{-1}) - (2.76 \times 10^7 \text{ m}^{-1})(26.97 \text{ V})}{5.74 \times 10^{14} \text{ m}^{-2}} \\
 &= 1.217 \times 10^{-6} \text{ V m}
 \end{aligned}$$

and the y-intercept

$$\begin{aligned}
 c' &= \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{D} \\
 &= \frac{(19.082 \times 10^{13} \text{ m}^{-2})(26.97 \text{ V}) - (2.76 \times 10^7 \text{ m}^{-1})(20.610 \times 10^7 \text{ V m}^{-1})}{(5.74 \times 10^{14} \text{ m}^{-2})} \\
 &= -0.944 \text{ V}
 \end{aligned}$$

The error on m is

$$s_m = \left[\frac{n\Sigma \delta^2}{(n-2)D} \right]^{\frac{1}{2}}$$

$$\begin{aligned}
 &= \left[\frac{(7)(13.48 \times 10^{-2} \text{ V}^2)}{(5)(5.74 \times 10^{14} \text{ m}^{-2})} \right]^{\frac{1}{2}} \\
 &= 1.81 \times 10^{-8} \text{ V m.}
 \end{aligned}$$

and the error on c is

$$\begin{aligned}
 s_c &= \left[\frac{\Sigma \delta^2 \Sigma x^2}{(n-2)D} \right]^{\frac{1}{2}} \\
 &= \left[\frac{(13.48 \times 10^{-2} \text{ V}^2)(19.082 \times 10^{13} \text{ m}^{-2})}{(5)(5.74 \times 10^{14} \text{ m}^{-2})} \right]^{\frac{1}{2}} \\
 &= 9.47 \times 10^{-2} \text{ V}
 \end{aligned}$$

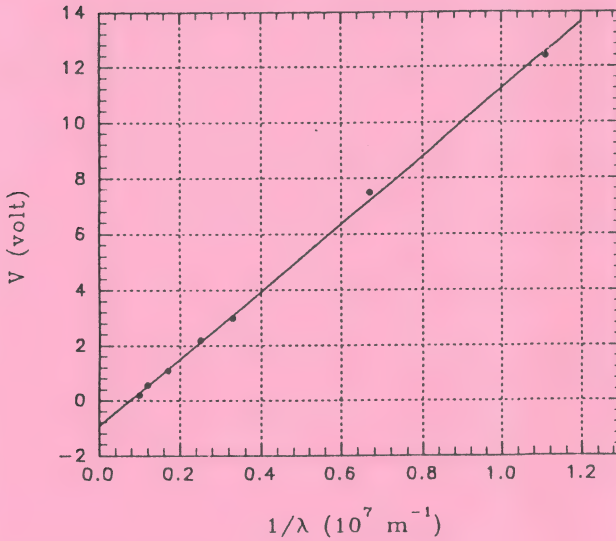


Figure 6: Graph of V versus $\frac{1}{\lambda}$.

Planck's constant follows from the slope

$$\begin{aligned}
 h &= \frac{mc}{e} \\
 &= \frac{(1.217 \times 10^{-6} \text{ V m})(1.6 \times 10^{-19} \text{ C})}{(2 \times 10^8 \text{ m s}^{-1})} \\
 &= 6.491 \times 10^{-34} \text{ Js}
 \end{aligned}$$

and the error on h is given by

$$\begin{aligned}
 \Delta h &= \frac{\Delta me}{c} \\
 &= \frac{s_m e}{c} \\
 &= \frac{(1.81 \times 10^{-8} \text{ V m})(1.6 \times 10^{-19} \text{ C})}{(3 \times 10^8 \text{ m s}^{-1})} \\
 &= 9.65 \times 10^{-36} \text{ Js.}
 \end{aligned}$$

Thus

$$\begin{aligned}
 h &= (6.491 \pm 0.0965) \times 10^{-34} \text{ Js} \\
 &\simeq (6.5 \pm 0.1) \times 10^{-34} \text{ Js}
 \end{aligned}$$

Note that the known value of $h = 3.63 \times 10^{-34} \text{ Js}$ falls outside the error margin of $6.40 \times 10^{-34} \text{ Js}$ to $6.6 \times 10^{-34} \text{ Js}$ which was obtained experimentally. This indicates that a systematic error was made in obtaining the data.

The work function of the metal follows from the y-intercept

$$\begin{aligned}
 \phi &= ec' \\
 &= -(1.6 \times 10^{-19} \text{ C})(-0.944 \text{ V}) \\
 &= 1.510 \times 10^{-19} \text{ V}
 \end{aligned}$$

and the error on ϕ is given by

$$\begin{aligned}
 \Delta \phi &= e \Delta c' \\
 &= e s_c \\
 &= (1.6 \times 10^{-19} \text{ C})(9.47 \times 10^{-2} \text{ V}) \\
 &= 1.515 \times 10^{-20} \text{ V}
 \end{aligned}$$

Thus

$$\begin{aligned}
 \phi &= (1.510 \pm 0.1515) \times 10^{-19} \text{ V} \\
 &\simeq (1.5 \pm 0.2) \times 10^{-19} \text{ V.}
 \end{aligned}$$

The threshold frequency is

$$f = \frac{\phi}{h}$$

$$\begin{aligned}
 &= \frac{(1.51 \times 10^{-19} \text{ V})}{(6.5 \times 10^{-34} \text{ Js})} \\
 &= 2.32 \times 10^{14} \text{ Hz}
 \end{aligned}$$

and the error on f is given by

$$\frac{\Delta f}{f} = \left[\left(\frac{\Delta \phi}{\phi} \right)^2 + \left(\frac{\Delta \phi}{h} \right)^2 \right]^{\frac{1}{2}}$$

so

$$\begin{aligned}
 \Delta f &= (2.32 \times 10^{14} \text{ Hz}) \left[\left(\frac{0.2}{1.5} \right)^2 + \left(\frac{0.1}{6.4} \right)^2 \right]^{\frac{1}{2}} \\
 &= 3.57 \times 10^{12} \text{ Hz}.
 \end{aligned}$$

Thus

$$\begin{aligned}
 f &= (2.32 \pm 0.0357) \times 10^{14} \text{ Hz} \\
 &\simeq (2.32 \pm 0.04) \times 10^{14} \text{ Hz}.
 \end{aligned}$$

[25]

Post Mortem for Assignment 3

Dear Student

I am glad to be able to say that everybody now seems to be coming to grips with this "problem module" of ours. The work submitted for Assignment 03 was considerably better than for Assignments 01 and 02.

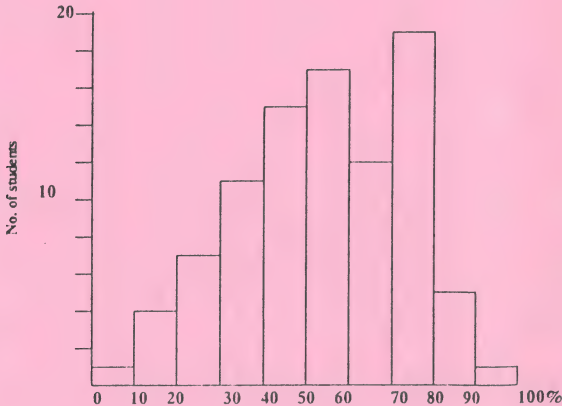


Fig. 1: Histogram of marks obtained in Assignment 03.

In addition to the solutions in the key, I would like to make the following comments:

9.2

From the answers we received to this question, *linear circuit element* is clearly an unknown term. The term does NOT refer to the way (i.e. series or parallel) in which the circuit element is connected to the circuit! *Linear* in this case refers to the relation between the current I and the potential difference V across the circuit element, i.e. a *direct proportionality* as is shown by the graph of I versus V in Figure 2(a). The resistance R is constant in this case. Such a circuit element obeys *Ohm's law*, and is sometimes referred to as an *Ohmic* circuit element.

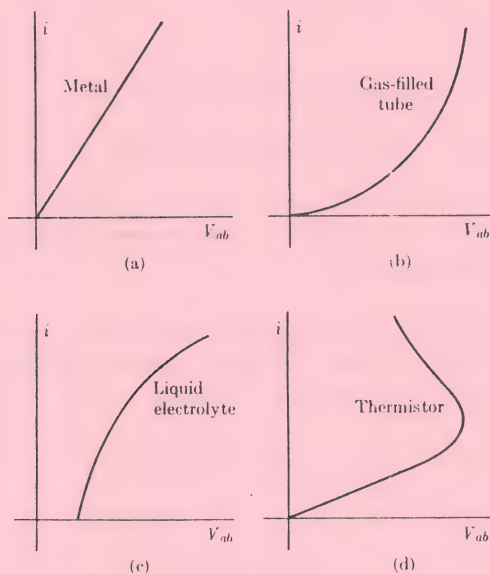


Fig. 2: Four examples of the relation between current and potential difference. (a) A metal for which R is constant, (b) A vacuum tube, where R decreases, (c) An electrolyte, where R first decreases and then increases, (d) A thermistor, where R is first constant, and then decreases. (Richards et al. 1962 *Modern College Physics*. Addison Wesley: Reading, Massachusetts).

The graphs in Figure 2(b), (c) and (d) illustrate the behaviour of *nonlinear conductors* where resistance is not constant.

9.21

The mistake that occurred most frequently in the answers to this problem, was that the dependent and independent variables were interchanged. Remember, the *dependent variable* I is measured as a function of the *independent variable*, V in this case. That means that the potential difference across the resistor was changed (the person executing the experiment *sets* V at different values) and I is then measured for each value of V , i.e. the value of I *depends* on the value of V .

10.12

This question was misunderstood by a large number of students. By *reaction* to the force $\vec{F}$ is meant the *reaction force* to which Newton's third law refers: "For every action there is an equal and opposite reaction".

10.23

This problem constitutes an example of an equation for which it is not possible to separate the variables, i.e. the quantities we plot on the x-axis in this case, $x^2 + y^2$, contains the dependent variable y and is therefore not independent of y . For this reason, no linear regression analysis was performed on the data.

11.5

Many students were not aware of the fact that the torque on a current carrying loop in a magnetic field is given by

$$\tau = IA \hat{n} \times \vec{B}$$

Please remember that the book *Principles of Physics* by Bueche should also be consulted with regards to the theory covered by this module.

11.14

Please note that the equation for this question in the Study Guide does not show the velocity $\vec{v}$ as a vector, which is incorrect. The equation should read

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}.$$

11.16

Units proved to be a general stumbling block in this problem. Many students indicated the unit of magnetic field strength B to be newton, which is of course, incorrect. The unit for magnetic field strength is the tesla, which is defined by

$$1 \text{ T} = 1 \text{ N A}^{-1} \text{ m}^{-1}.$$

This definition follows from the fact that the force on a moving charge in a magnetic field is given by

$$\vec{F} = q\vec{v} \times \vec{B}.$$

The magnitude of this force is therefore

$$F = qvB \sin \theta$$

where θ is the angle between $\vec{v}$ and $\vec{B}$ and therefore

$$B = \frac{F}{qv \sin \theta} .$$

Clearly the unit of B is given by

$$\frac{\text{newton}}{(\text{coulomb})(\text{meter second}^{-1})}$$

and because $1 \text{ A} = 1 \text{ C s}^{-1}$, this simplifies to

$$1 \text{ T} = 1 \frac{\text{N}}{\text{A m}} = 1 \text{ N A}^{-1} \text{ m}^{-1} .$$

I hope that these comments have helped! Good luck with the last assignment - we're almost there.

Kind regards

MRS J A CILLIERS

Tel.: 429-8018

UNISA

